Course of Theoretical Physics
LEV DAVIDOVICH LANDAU
Reading Notes

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列夫·达维多维奇·朗道（1908—1968） 理论物理学家、苏联科学院院士、诺贝尔物理学奖获得者。1908 年 1 月 22 日生于今阿塞拜疆共和国的首都巴库，父母是工程师和医生。朗道 19 岁从列宁格勒大学物理系毕业后在列宁格勒物理技术研究所开始学术生涯。1929—1931 年赴德国、瑞士、荷兰、英国、比利时、丹麦等国家进修，特别是在哥本哈根，曾受益于玻尔的指引。1932—1937 年，朗道在哈尔科夫担任乌克兰物理技术研究所理论部主任。从 1937 年起在莫斯科担任苏联科学院物理问题研究所理论部主任。朗道非常重视教学工作，曾先后在哈尔科夫大学、莫斯科大学等学校教授理论物理，撰写了大量教材和科普读物。

朗道的研究工作几乎涵盖了从流体力学到量子场论的所有理论物理学分支。1927 年朗道引入量子力学中的重要概念——密度矩阵；1930 年创立电子抗磁性的量子理论（相关现象被称为朗道抗磁性，电子的相应能级被称为朗道能级）；1935 年创立铁磁性的磁畴理论和反铁磁性的理论解释；1936—1937 年创立二级相变的一般理论和超导体的中间态理论（相关理论被称为朗道相变理论和朗道中间态结构模型）；1937 年创立原子核的概率理论；1940—1941 年创立液氦的超流理论（被称为朗道超流理论）和量子液体理论；1946 年创立等离子体振动理论（相关现象被称为朗道阻尼）；1950 年与金兹堡一起创立超导理论（金兹堡－朗道唯象理论）；1954 年创立基本粒子的电荷约束理论；1956—1958 年创立了费米液体的量子理论（被称为朗道费米液体理论）并提出了弱相互作用的 CP 不变性。

朗道于 1946 年当选为苏联科学院院士，曾 3 次获得苏联国家奖；1954 年获得社会主义劳动英雄称号；1961 年获得马克斯·普朗克奖章和弗里茨·伦敦奖；1962 年他与栗弗席兹合著的《理论物理学教程》获得列宁奖，同年，他因为对凝聚态物质特别是液氦的开创性工作而获得了诺贝尔物理学奖。朗道还是丹麦皇家科学院院士、荷兰皇家科学院院士、英国皇家学会会员、美国国家科学院院士、美国国家艺术与科学院院士、英国和法国物理学会的荣誉会员。

“朗道十诫”石板*

1958年苏联原子能研究所为庆贺朗道50岁寿辰，送给他的刻有朗道在物理学上最重要的10项科学成果的大理石板，这10项成果是：

1. 量子力学中的密度矩阵和统计物理学（1927年）
2. 自由电子抗磁性的理论（1930年）
3. 二级相变的研究（1936—1937年）
4. 铁磁性的磁畴理论和反铁磁性的理论解释（1935年）
5. 超导体的混合态理论（1934年）
6. 原子核的概率理论（1937年）
7. 氦Ⅱ超流性的量子理论（1940—1941年）
8. 基本粒子的电荷约束理论（1954年）
9. 费米液体的量子理论（1956年）
10. 弱相互作用的CP不变性（1957年）

* Бессараб М Я. Ландау: Страницы жизни. Москва: Московский рабочий, 1988.

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Pxi: He devoted himself so strenuously that often he became so exhausted that at night he could not sleep, still turning over formulae in his mind.

Later he used to describe how at that time he was amazed by the incredible beauty of the general theory of relativity (sometimes he even would declare that such a rapture on first making one's acquaintance with this theory should be a characteristic of any born theoretical physicist).
He also described the state of ecstasy to which he was brought on reading the articles by Heisenberg and Schrodinger signaling the birth of the new quantum mechanics. He said that he derived from them not only delight in the true glamour of science but also an acute realization of the power of the human genius, whose greatest triumph is that man is capable of apprehending things beyond the pale of his imagination. And of course, the curvature of space-time and the uncertainty principle are precisely of this kind.

As early as 1927, he done a study of the problem of damping in quantum mechanics, which first introduced a description of the state of a system with the aid of the density matrix.

The changes which occurred in him with the years and transformed him into a buoyant and gregarious individual were largely a result of his characteristic self-discipline and feeling of duty toward himself. These qualities, together with his sober and self-critical mind, enabled him to train himself and to evolve into a person with a rare ability — the ability to be happy. The same sobriety of mind enabled him always to distinguish between what is of real value in life and what is unimportant triviality, and thus also to retain his mental equilibrium during the difficult moments which occurred in his life too.

In 1929, at the Institute of Theoretical Physics, theoretical physicists from all Europe gathered round the great Niels Bohr and, during the famous seminars headed by Bohr, discussed all the basic problems of the theoretical physics of the time. This scientific atmosphere, enhanced by the charm of the personality of Bohr himself, decisively influenced Landau in forming his own outlook on physics and subsequently he always considered himself a disciple of Niels Bohr. He visited Copen- hagen two more times, in 1933 and 1934. Landau's sojourn abroad was the occasion, in particular, of his work on the theory of the diamagnetism of an electron gas [4] and the study of the limitations imposed on the measurability of physical quantities in the. relativistic quantum region (in collaboration with Peierls) [6].

Twentieth-century theoretical physics is rich in illustrious names of trail-blazing creators, and Landau was one of these creators. But his influence on scientific progress was far from exhausted by his personal contribution to it. He was not only an outstanding physicist but also a genuinely outstanding educator, a born educator. In this respect one may take the liberty of comparing Landau only to his own teacher — Niels Bohr.

Landau always attached great importance to the mastering of mathematical techniques by the theoretical physicist. 'The degree of this mastery should be such that, insofar as possible, mathematical complications would not distract attention from the physical difficulties of the problem — at least whenever standard mathematical techniques are concerned. 'This can be achieved only by sufficient training. Yet experience shows that the current style and programs for university instruction in mathematics for physicists often do not ensure such training. Experience also shows that after a physicist commences his independent research activity he finds the study of mathematics too "boring"'.

Therefore, the first test which Landau gave to anyone who desired to become one of his students was a quiz in mathematics in its '"practical" calculational aspects.
(The requirements were: ability to evaluate any indefinite integral that can be expressed in terms of elementary functions and to solve any ordinary differential equation of the standard type, knowledge of vector analysis and tensor algebra as well as of the principles of the theory of functions of a complex variable (theory of residues, Laplace method). It was assumed that such fields as tensor analysis and group theory would be studied together with the fields of theoretical physics to which they apply.)
The successful applicant could then pass on to the study of the seven successive sections of the program for the "theoretical minimum', which includes basic knowledge of all the domains of theoretical physics, and subsequently take an appropriate examination. In Landau's opinion, this basic knowledge should be mastered by any theoretician regardless of his future specialization. Of course, he did not expect anyone to be as universally well-versed in science as he himself. But he thus manifested his belief in the integrity of theoretical physics as a single science with unified methods.

In a remarkable interaction with experimental research, Landau created what may be the outstanding accomplishment of his scientific life — the theory of quantum fluids.

Lastly, in 1962 he was awarded the Nobel Prize in Physics 'for his pioneering theories for condensed matter, especially liquid helium'

Landau's scientific influence was, of course, far from confined to his own disciples. He was deeply democratic in his life as a scientist (and in his life as a human being, for that matter; pomposity and deference to titles always remained foreign to him). Anyone, regardless of his scientific merits and title, could ask Landau for counsel and criticism (which were invariably precise and clear), on one condition only: the question must be businesslike instead of pertaining to what he detested most in science: empty Philosophizing or vapidity and futility cloaked in pseudo-scientific sophistries. He had an acutely critical mind; this quality, along with his approach from the standpoint of profound physics, made discussion with him extremely attractive and useful.

In discussion he used to be ardent and incisive but not rude; witty and ironic but not caustic. The nameplate which he hung on the door of his office at the Ukrainian Physicotechnical Institute bore the inscription:

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L. LANDAU
BEWARE, HE BITES!
With years his character and manner mellowed somewhat, but his enthusiasm
for science and his uncompromising attitude toward science remained unchanged.
And certainly his sharp exterior concealed a scientifically impartial
attitude, a great heart and great kindness.  However harsh and unsparing
he may have been in his critical comments, he was just as intense in
his desire to contribute with his advice to another man's success, and
his approval, when he gave it, was just as ardent.
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These traits of Landau's personality as a scientist and of his talent actually elevated him to the position of a supreme scientific judge, as it were, over his students and colleagues.

† This position is symbolized in A. A. Yuzefovich's well-known friendly cartoon, "Dau said", reproduced here.

There is no doubt that this side of Landau's activities, his scientific and moral authority which exerted a restraining influence on frivolity in research, has also markedly contributed to the lofty level of our theoretical physics.
His constant scientific contact with a large number of students and colleagues also represented to Landau a source of knowledge. A unique aspect of his style of work was that, ever since long ago, since the Khar'kov years, he himself almost never read any scientific article or book but nevertheless he was always completely au courant with the latest news in physics.

He derived this knowledge from numerous discussions and from the papers presented at the seminar held under his direction. ‘ This seminar was held regularly once a week for nearly 30 years, and in the last years its sessions became gatherings of theoretical physicists from all Moscow. ‘The presentation of papers at this seminar became a sacred duty for all students and co-workers, and Landau himself was extremely serious and thorough in selecting the material to be presented. He was interested and equally competent in every aspect of physics and the participants in the seminar did not find it easy to follow his train of thought in instantaneously switching from the discussion of, say, the properties of "strange" particles to the discussion of the energy spectrum of electrons in silicon. ‘l'o Landau himself listening to the papers was never an empty formality: he did not rest until the essence of a study was completely elucidated and all traces of "philology" — unproved statements or propositions made on the principle of "why might it not" — therein were eliminated. As a result of such discussion and criticism many studies were condemned as "pathology" and Landau completely lost interest in them. On the other hand, articles that really contained new ideas or findings were included in the so-called "gold fund" and remained in Landau's memory for ever. In fact, usually it was sufficient for him to know just the guiding idea of a study in order to reproduce all of its findings. As a rule, he found it easier to obtain them on his own than to follow in detail the author's reasoning. In this way he reproduced for himself and profoundly thought out most of the basic results obtained in all the domains of theoretical physics. This probably also was the reason for his phenomenal ability to answer practically any question concerning physics that might be asked of him.

Landau's scientific style was free of the — unfortunately fairly widespread — tendency to complicate simple things (often on the grounds of generality and rigour which, however, usually turn out to be illusory). He himself always strove towards the opposite — to simplify complex things, to uncover in the most lucid manner the genuine simplicity of the laws underlying the natural phenomena. This ability of his, this skill at "trivializing" things as he himself used to say, was to him a matter of special pride.
The striving for simplicity and order was an inherent part of the structure of Landau's mind. It manifested itself not only in serious matters but also in semi-serious things as well as in his characteristic personal sense of humor. Thus, he liked to classify everyone, from women according to the degree of their beauty, to theoretical physicists according to the significance of their contribution to science. This last classification was based on a logarithmic scale of five: thus, a second-class physicist supposedly accomplished 10 times as much as a third-class physicist ("pathological types" were ranked in the fifth class). On this scale Einstein occupied the position $\frac{1}{2}$, while Bohr, Heisenberg, Schrodinger, Dirac and certain others were ranked in the first class. Landau modestly ranked himself for a long time in class $2\frac{1}{2}$ and it was only comparatively late in his life that he promoted himself to the second class.

He always worked hard (never at a desk, usually reclining on a divan at home).

At any rate there is no doubt that his drive for work was inherently motivated not by desire for fame but by an inexhaustible curiosity and passion for exploring the laws of nature in their large and small manifestations. He never omitted a chance to repeat the elementary truth that one should never work for extraneous purposes, work merely for the sake of making a great discovery, for then nothing would be accomplished anyway.

The range of Landau's interests outside physics also was extremely wide. In addition to the exact sciences he loved history and was well-versed in it. He was also passionately interested in and deeply impressed by every genre of fine arts, though with the exception of music (and ballet).
Those who had the good fortune to be his students and friends for many years knew that our Dau, as his friends and comrades nicknamed him, did not grow old. In his company boredom vanished. The brightness of his personality never grew dull and his scientific power remained strong. All the more senseless and frightful was the accident which put an end to his brilliant activity at its zenith.

Landau's articles, as a rule, display all the features of his characteristic scientific style: clarity and lucidity of physical statement of problems, the shortest and most elegant path towards their solution, no superfluities. Even now, after many years, the greater part of his articles does not require any revisions.

A characteristic feature of Landau's scientific creativity is its almost unprecedented breadth, which encompasses the whole of theoretical physics, from hydrodynamics to the quantum field theory. In our century, which is a century of increasingly narrow specialization, the scientific paths of his students also have been gradually diverging, but Landau himself unified them all, always retaining a truly astounding interest in everything. It may be that in him physics has lost one of the last great universalists.

These include, in the first place, several of his early works. In the course of his studies of the radiation-damping problem he was the first to introduce the concept of incomplete quantum-mechanical description accomplished with the aid of quantities which were subsequently termed the density matrix [2]. In this article the density matrix was introduced in its energy representation.

Of no smaller virtuosity was Landau's work dealing with the elaboration of Fermi's idea of the statistical nature of multiple particle production in collisions [74]. This study also represents a brilliant example of the methodological unity of theoretical physics in which the solution of a problem is accomplished by using the methods from a seemingly completely different domain. Landau showed that the process of multiple production includes the stage of the expansion of a "cloud" whose dimensions are large com- pared with the mean free path of particles in it; correspondingly, this stage should be described by equations of relativistic hydrodynamics. The solution of these equations required a number of ingenious techniques as well as a thorough analysis. Landau used to say that this study cost him more effort than any other problem that he had ever solved.

According to classical mechanics and statistics, a change in the pattern of movement of free electrons in a magnetic field cannot result in the appearance of new magnetic properties of the system. Landau was the first to elucidate the character of this motion in a magnetic field for the quantum case, and to show that quantization completely changes the situation, resulting in the appearance of diamagnetism of the free electron gas ("Landau diamagnetism" as this effect is now termed) [4]. The same study qualitatively predicted the periodic dependence of the magnetic susceptibility on the intensity of the magnetic field when this intensity is high. At the time (1930) this phenomenon had not yet been observed by anyone, and it was experimentally discovered only later (the De Haas Van Alphen effect); a quantitative theory of this effect was presented by Landau in a later paper [38].

But the most significant contribution that physics owes to Landau is his theory of quantum liquids. The significance of this new discipline at present is steadily growing; there is no doubt that its development in recent decades has produced a revolutionary effect on other domains of physics as well — on solid-state physics and even on nuclear physics.

Concluding this brief and far from complete survey, it only remains to be repeated that to physicists there is no need to emphasize the significance of Landau's contribution to theoretical physics. His accomplishments are of lasting significance and will for ever remain part of science.

Chapter 1

The Equation of Motion

1.1 Generalized coordinates

The number of independent quantities which must be specified in order to define uniquely the position of any system is called the number of degrees of freedom.

1.2 The principle of least action

The most general formulation of the law governing the motion of mechanical systems is **the principle of least action or Hamilton's principle**, according to which every mechanical system is characterized by a definite function

$$L(q_1, q_2, ..., q_s, \dot{q}_1, \dot{q}_2, ..., \dot{q}_s, t),$$

or briefly $L(q, \dot{q}, t)$, and the motion of the system is such that a certain condition is satisfied.

Let the system occupy, at the instants t_1 and t_2, positions defined by two sets of values of the co-ordinates, $q^{(1)}$ and $q^{(2)}$. Then the condition is that the system moves between these positions in such a way that the integral

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)dt$$

takes the least possible value. The function L is called **Lagrangian(拉格朗日函数)** of the system concerned, and the integral is called the **action(作用量)**.
根据最小作用量原理推得(对推导过程有一个疑问：公式 2.3 为什么成立？原书有一句解释：Since, for $t = t_1$ and for $t = t_2$, all the functions 2.2 must take the

values $q^{(1)}$ and $q^{(2)}$ respectively, 不理解):

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - \frac{\partial L}{\partial q} = 0$$

When the system has more than one degree of freedom, the s different functions $q_i(t)$ must be varied independently in the principle of least action. We then evidently obtain s equations of the form

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} = 0$$

These are the required differential equations, called in mechanics **Lagrange's equations(拉格朗日方程)**(In the calculus of variations they are Euler's equations for the formal problem of determining the extrema of an integral of the form (2.1))

If the Lagrangian of a given mechanical system is known, the equations (2.6) give the relations between accelerations, velocities and coordinates, i.e. they are the equations of motion of the system.

Mathematically, the equations (2.6) constitute a set of s second-order equations for s unknown functions $q_i(t)$. The general solution contains 2s arbitrary constants. To determine these constants and thereby to define uniquely the motion of the system, it is necessary to know the initial conditions which specify the state of the system at some given instant, for example the initial values of all the coordinates and velocities.

Thus the Lagrangian is defined only to within an additive total time derivative of any function of co-ordinates and time. (可见，拉格朗日函数仅可以定义到相差一个对时间和坐标的任意 函数的时间全导数项.)

1.3 Galileo's relativity principle

The problem naturally arises of finding a frame of reference in which the laws of mechanics take their simplest form.
It is found, however, that a frame of reference can always be chosen in which space is homogeneous and isotropic and time is homogeneous. This is called an **inertial frame**. In particular, in such a frame a free body which is at rest at some instant remains always at rest.
接下来推导拉格朗日函数的具体形式：
We can now draw some immediate inferences concerning the form of the Lagrangian of a particle, moving freely, in an inertial frame of reference. The homogeneity of space and time implies that the Lagrangian cannot contain explicitly

either the radius vector r of the particle or the time t, i.e. L must be a function of the velocity v only. Since space is isotropic, the Lagrangian must also be independent of the direction of v, and is therefore a function only of its magnitude, i.e. of $\mathbf{v}^2 = v^2$:

$$L = \mathbf{L}(v^2)$$

Since the Lagrangian is independent of $\mathbf{r}$, we have $\partial L/\partial r = 0$, and so Lagrange's equation:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - \frac{\partial L}{\partial q} = 0$$

becomes:

$$\frac{d}{dt}(\frac{\partial L}{\partial v}) = 0,$$

whence $\partial L/\partial v = constant$.
Since $\partial L/\partial v$ is a function of the velocity only(从前面分析的惯性系的特点得出的结论), it follows that:

$$\mathbf{v} = constant.$$

Thus we conclude that, in an inertial frame, any free motion takes place with a velocity which is constant in both magnitude and direction. This is the law of inertia.

Experiment shows that not only are the laws of free motion the same in the two frames, but the frames are entirely equivalent in all mechanical re- spects. Thus there is not one but an infinity of inertial frames moving, relative to one another, uniformly in a straight line. In all these frames the properties of space and time are the same, and the laws of mechanics are the same. This constitutes Galileo's relativity principle, one of the most important principles of mechanics.

The assumption that time is absolute is one of the foundations of classical mechanics.(The assumption does not hold good in relative mechanics)

Galileo's relativity principle can be formulated as asserting the invariance of the mechanical equations of motion under any such transformation.(伽利略相对性原理可以表述为: 力学运动方程在伽利略变换下具有不变性.)

1.4 The Lagrangian for a free particle

中文翻译书里公式 (4.3) 有问题， 看半天不理解，回到英文版原书才发现翻译书里那样写不对.

1.5 The Lagrangian for a system of particles

为了描述封闭质点系内质点间的相互作用,可以在自由质点系的拉格朗日函数(4.2)中增加坐标的某一函数(根据相互作用的性质确定), 将这个函数记为 $-U$ (势能项在这里直接就加进来了，不理解原因)
知道拉格朗日函数后就可以建立运动方程——推导出牛顿方程
势能可以增减任意常数而不改变运动方程(这是在section2末讲到的拉格朗日函数不确定性的特殊情况).选择这个任意常数的最自然和最通用的方法是,当无限增大质点间距离时势能趋向于零.

1.6 Problems

Chapter 2

Conservation Laws

2.1 Energy

DURING the motion of a mechanical system, the $2s$ quantities q_i and $\dot{q}_i$; $(i = 1, 2, ..., s)$ which specify the state of the system vary with time. There exist, however, functions of these quantities whose values remain constant during the motion, and depend only on the initial conditions. Such functions are called **integrals of the motion.**
由时间的均匀性导出的守恒定律.
其中用到Euler's theorem on homogeneous functions,齐次函数的欧拉定理.

2.2 Momentum

A second conservation law follows from the homogeneity of space（空间的均匀性）. By virtue of this homogeneity, the mechanical properties of a closed system are unchanged by any parallel displacement of the entire system in space. Let us therefore consider an infinitesimal displacement ϵ, and obtain the condition for the Lagrangian to remain unchanged.

这两节分别从时间和空间的均匀性出发，由基本的拉格朗日方程推导出能量守恒和动量守恒原理。而时间和空间的均匀性又是由前面对惯性参考系的选取的讨论得到的结论。后面由空间的各向同性，推导出了角动量守恒定律。

In particular, for a system of only two particles, $F_1 + F_2 = 0$: the force exerted by the first particle on the second is equal in magnitude, and opposite in direc- tion, to that exerted by the second particle on the first. This is the equality of action and reaction (Newton's third law).
到此，牛顿三定律，还有自由运动质点系统的能量守恒和动量守恒定理全部由

拉格朗日方程和数学工具推导出来!

If the motion is described by generalised co-ordinates q_i, the derivatives of the Lagrangian with respect to the generalised velocities

$$p_i = \partial L/\partial \dot{q}_i$$

are called generalised momenta, and its derivatives with respect to the generalised co-ordinates

$$F_i = \partial L/\partial q_i$$

are called generalised forces. In this notation, Lagrange's equations are

$$\dot{p}_i = F_i$$

In Cartesian co-ordinates the generalised momenta are the components of the vectors P_a. In general, however, the p_i are linear homogeneous functions of the generalised velocities $\dot{q}_i$, and do not reduce to products of mass and velocity.

2.2.1 Problem

2.3 Center of mass

If the total momentum of a mechanical system in a given frame of reference is zero, it is said to be at rest relative to that frame. This is a natural generalisation of the term as applied to a particle. Similarly, the velocity V given by (8.2) is the velocity of the "motion as a whole" of a mechanical system whose momentum is not zero. Thus we see that the law of conservation of momentum makes possible a natural definition of rest and velocity, as applied to a mechanical system as a whole.

The law of conservation of momentum for a closed system can be formulated as stating that the centre of mass of the system moves uniformly in a straight line. In this form it generalises the law of inertia derived in §3 for a single free particle, whose "centre of mass" coincides with the particle itself.

2.3.1 Problem

Find the law of transformation of the action S from one inertial frame to another.

Sorurion: The Lagrangian is equal to the difference of the kinetic and potential energies, and is evidently transformed in accordance with a formula analogous to (8.5):

$$L = L' + \mathbf{V} \cdot \boldsymbol{P'} + \frac{\mu V^2}{2}$$

Integrating this with respect to time, we obtain the required law of transformation of the action:

$$S = S' + \mu V \cdot R' + \frac{\mu V^2}{2} t$$

where R' is the radius vector of the centre of mass in the frame K'.

2.4 Angular momentum

Let us now derive the conservation law which follows from the isotropy of space（空间的各项同性）. This isotropy means that the mechanical properties of a closed system do not vary when it is rotated as a whole in any manner in space. Let us therefore consider an infinitesimal(分析这类问题惯用的思路) rotation of the system, and obtain the condition for the Lagrangian to remain unchanged.

There are no other additive integrals of the motion. Thus every closed system has seven such integrals: energy, three components of momentum, and three components of angular momentum.

2.5 Mechanical similarity

Multiplication of the Lagrangian by any constant clearty does not affect the equations of motion. This fact (already mentioned before) makes possible, in a number of important cases, some useful inferences concerning the properties of the motion, without the necessity of actually integrating the equations.
Such cases include those where the potential energy is a homogeneous function of the co-ordinates, i.e. satisfies the condition

$$U\left(\alpha r_1, \alpha r_2, \cdots, \alpha \boldsymbol{r}_n\right) = \alpha^k U\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_n\right)$$

where α is any constant and k the degree of homogeneity of the function.

Let us carry out a transformation in which the co-ordinates are changed by a factor α and the time by a factor β:

$$\mathbf{r}_a \rightarrow \alpha \mathbf{r}_a, t \rightarrow \beta t$$

. Then all the velocities $\mathbf{v}_a = \mathrm{d}\mathbf{r}_a/\mathrm{d}t$ are changed by a factor α/β, and the kinetic energy by a factor α^2/β^2. The potential energy is multiplied by α^k. If α and β are such that $\alpha^2/\beta^2 = \alpha^k$, ie. $\beta = \alpha^{1-\frac{1}{2}k}$, then the result of the transformation is to multiply the Lagrangian by the constant factor α^k, i.e. to leave the equations of motion unaltered.

A change of all the co-ordinates of the particles by the same factor corresponds to the replacement of the paths of the particles by other paths, geometrically similar but differing in size. Thus we conclude that, if the potential energy of the system is a homogeneous function of degree k in the (Cartesian) co- ordinates, the equations of motion permit a series of geometrically similar paths, and the times of the motion between corresponding points are in the ratio

$$\frac{t'}{t} = \left(\frac{l'}{l}\right)^{1-k/2}$$

where l'/l is the ratio of linear dimensions of the two paths. Not only the times but also any mechanical quantities at corresponding points at corresponding times are in a ratio which is a power of l'/l. For example, the velocities, energies and angular momenta are such that

$$v'/v = (l'/l)^{\frac{1}{2}k}, \quad E'/E = (l'/l)^{k}, \quad M'/M = (l'/l)^{1+\frac{1}{2}k}$$

The following are some examples of the foregoing.

As we shall see later, in small oscillations the potential energy is 2 quadratic function of the co-ordinates ($k = 2$). From (10.2) we find that the period of such oscillations is independent of their amplitude.

In a uniform field of force, the potential energy is a lincar function of the co-ordinates (see (5.8)), ie. $k = 1$. From (10.2) we have $t'/t = \sqrt{(l'/l)}$, Hence, for example, it follows that, in fall under gravity, the time of fall is as the square root of the initial altitude.

In the Newtonian attraction of two masses or the Coulomb interaction of two charges, the potential energy is inversely proportional to the distance apart, i.e. it is a homogeneous function of degree $k = -1$. Then $t'/t = (l'/l)^{3/2}$, and we can state, for instance, that the square of the time of revolu- tion in the orbit is as the cube of the size of the orbit (Kepler's third law).

If the potential energy is a homogeneous function of the co-ordinates and the motion takes place in a finite region of space, there is a very simple relation between the time average values of the kinetic and potential energies, known as the vrial theorem.

$$\overline{U} = \frac{2}{k+2}E, \quad \overline{T} = \frac{k}{k+2}E$$

which express $\overline{U}$ and $\overline{T}$ in terms of the total energy of thesystem.

In particular, for small oscillations ($k = 2$) we have $\overline{T} = \overline{U}$, i.e. the mean values of the kinetic and potential energies are equal. For a Newtonian interaction ($k = -1$)$2\overline{T} = -\overline{U}$, and $E = -\overline{T}$, in accordance with the fact that, in such an interaction, the motion takes place in a finite region of space only if the total energy is negative.

2.5.1 Problems

Chapter 3

Interaction of the equations of motions

3.1 Motion in one dimension

3.1.1 Problems

这几题主要就是套

$$t=\sqrt{\frac{m}{2}}\int\frac{\mathrm{d}x}{\sqrt{E-U(x)}}+const$$

这个公式，而此公式又是由

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\sqrt{\frac{2}{m}[E-U(x)]}$$

积分而来，

$$\frac{m\dot{x}^2}{2}+U(x)=E$$

$$L=\frac{m\dot{x}^2}{2}-U(x)$$

3.2 Determination of the potential energy from the period of oscillation

Let us consider to what extent the form of the potential energy $U(x)$ of a field in which a particle is oscillating can be deduced from a knowledge of the period of oscillation T as a function of the energy E. Mathematically, this involves the solution of the integral equation (11.5), in which $U(x)$ is regarded as unknown and $T(E)$ as known.

3.3 The reduced mass

A complete general solution can be obtained for an extremely important problem, that of the motion of a system consisting of two interacting particles (the two-body problem).

As a first step towards the solution of this problem, we shall show how it can be considerably simplified by separating the motion of the system into the motion of the centre of mass and that of the particles relative to the centre of mass.

3.4 Motion in a central field

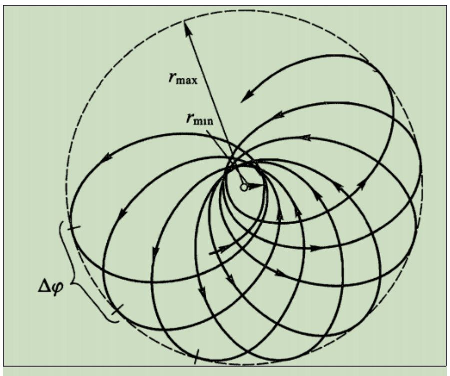
Any generalised co-ordinate g; which does not appear explicitly in the Lagrangian—is said to be cyclic.(拉格朗日函数不显含的广义坐标称为循环坐标)

This law has a simple geometrical interpretation in the plane motion of a single particle in a central field. The expression $\frac{1}{2}r \cdot rd\phi$is the area of the sector bounded by two neighbouring radius vectors and an element of the path. Calling this area df, we can write the angular momentum of the particle as

$$M = 2m\dot{f}$$

where the derivative $\dot{f}$ is called the sectorial velocity. Hence the conservation of angular momentum implies the constancy of the sectorial velocity: in equal times the radius vector of the particle sweeps out equal areas (Kepler's second law).

In general, therefore, the path of a particle executing a finite motion is not closed. It passes through the minimum and maximum distances an infinity of times, and after infinite time it covers the entire annulus between the two bounding circles. The path shown in the following is an example.



There are only two types of central field in which all finite motions take place in closed paths. They are those in which the potential energy of the particle varies as $1/r$ or as r^2. The former case is discussed in section 15; the latter is that of the space oscillator (see section 23, Problem 3).

3.4.1 Problems

3.5 Kepler's problem

An important class of central fields is formed by those in which the poten- tial energy is inversely proportional to r, and the force accordingly inversely proportional to r^2. They include the fields of Newtonian gravitational attraction and of Coulomb electrostatic interaction; the latter may be either attractive or repulsive.

$$\varphi = \int \frac{(M/r^2)\,\mathrm{d}r}{\sqrt{2m[E-U(r)]-M^2/r^2}} + \text{const}$$

$$U = -\alpha/r$$

将 U 代入得

$$\varphi = \arccos \frac{M/r - m\alpha/M}{\sqrt{2mE + m^2\alpha^2/M^2}} +$$

选择合适的起点使常数项为0，并引入记号

$$p = \frac{M^2}{m\alpha}, \quad e = \sqrt{1 + \frac{2EM^2}{m\alpha^2}}$$

轨道方程可重新写为：

$$p/r = 1 + e\cos\varphi$$

This is the equation of a conic section with one focus at the origin; $2p$ is called the latus rectum of the orbit and e the eccentricity. Our choice of the origin of Φ is seen from (15.5) to be such that the point where $\Phi = 0$ is the point ncarest to the origin (called the perihelion).

It is seen from (15.4) that, if $E < 0$, then the eccentricity $e < 1$, i.e. the orbit is an ellipse (Fig. 11) and the motion is finite, in accordance with what has been said earlier in this section.

3.5.1 Problems

Chapter 4

Collisions Between Particles

4.1 Disintegration of particles

4.2 Elastic collisions

4.3 Scattering

As already mentioned in last section, a complete calculation of the result of a collision between two particles (i.e. the determination of the angle χ) requires the solution of the equations of motion for the particular law of interaction involved.

散射截面的定义

4.3.1 Problems

The scattering is isotropic in the C system. On integrating do over all angles, we find that the total cross-section $\sigma = \pi a^2$, in accordance with the fact that the "impact area" which the particle must strike in order to be scattered is simply the cross-sectional area of the sphere.

4.4 Rutherford's formula

One of the most important applications of the formulae derived above is to the scattering of charged particles in a Coulomb field. Putting in (18.4) $U = \alpha/r$ and effecting the elementary integration, we obtain

$$\varphi_0 = \arccos \frac{\frac{\alpha}{mv_\infty^2 \rho}}{\sqrt{1 + \left(\frac{\alpha}{mv_\infty^2 \rho}\right)^2}}$$

Following is the Rutherford's formula:

$$d\sigma = \pi \left(\frac{\alpha}{mv_\infty^2}\right)^2 \frac{\cos\left(\frac{\chi}{2}\right)}{\sin^3\left(\frac{\chi}{2}\right)} d\chi$$

which gives the effective cross-section in the frame of reference which the centre of mass of the colliding particles is at rest.

4.4.1 Problems

4.5 Small-angle scattering

4.5.1 Problems

Chapter 5

Small Oscillations

5.1 Free oscillations in one dimension

Stable equilibrium corresponds to a position of the system in which its potential energy $U(q)$ is a minimum. A movement away from this position results in the setting up of a force $-dU/dq$ which tends to return the system to equilibrium.

The use of exponential factors is mathematically simpler than that of trigonometrical ones because they are unchanged in form by differentiation. So long as all the operations concerned are linear (addition, multiplication by constants, differentiation, integration), we may omit the sign re throughout and take the real part of the final result.

5.1.1 Problems

5.2 Forced oscillations

5.2.1 Problems

5.3 Oscillations of systems with more than one degree of freedom

5.3.1 Problems

5.4 Vibrations of molecules

If we have a system of interacting particles not in an external ficld, not all of its degrees of freedom relate to oscillations. A typical example is that of molecules. Besides motions in which the atoms oscillate about their positions of equilibrium in the molecule, the whole molecule can execute translational and rotational motions.

In solving a mechanical problem of molecular oscillations, it is convenient to eliminate immediately the translational and rotational degrees of freedom.

5.4.1 Problems

5.5 Damped oscillations