Tools of Radio Astronomy
Thomas L. Wilson, Sixth Edition
Reading Notes

Renkun Kuang

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Preface to the Sixth Edition

This book introduces the tools needed to pursue research in radio astronomy. These consist of:

1. The basic principles in Chaps. 1–4 and 6
2. The properties and uses of various types of receivers and antennas in Chaps. 5, 7, 8, and 9
3. The analysis of mechanisms responsible for broadband continuum and spectral line radiation in Chaps. 10–16

The aim of the author is to help users to **understand and apply well-established results**.

This book **uses relations and expressions in the form normally encountered in radio astronomy publications** so that the reader can **connect these with concepts found in basic physics and chemistry texts**.

There have been many major new instrumental developments since the fifth edition. One example is **the practical use of Focal Plane Arrays of receivers**. Another is **the Expanded Very Large Array (now the Jansky-VLA)**, and, more generally,interferometry at frequencies below $\nu = 1\text{GHz}$ and above $\nu = 200\text{GHz}$.

Over the past 30 years, the most dramatic improvements in interferometric aperture synthesis imaging arise from the use of faster computers and sophisticated image correction algorithms.

In modern systems, computer soft- and firmware has become more and more integrated with receiver hardware, so are an integral part of radio astronomical systems.

Other additions are the inclusion of the conventional derivations of receiver noise (Appendix C for Chap. 4) and the radiation field of a filled aperture (Appendix E for Chap. 6).

It is assumed that the readers have a thorough knowledge of physics. However, difficulties have often arisen when the instrumental topics were discussed. Often there is a difference between how a subject is treated in basic physics books and the way it is presented using the language of radio astronomy.

As to nomenclature, in the text and index, we refer to single radio telescopes as "antennas" and arrays of antennas with coupled outputs as "interferometers"; the more specialized instruments, such as those used to study the cosmic microwave background, are referred to as "facilities."

For each chapter, a list of references is given. Usually this list has two parts: (1) general references that are review articles and books that cover the general aspects and which often give an overview of the subjects covered and (2) special references for specific topics.

The book is intended to aid in applying the principles of radio astronomy, but it is not intended to be a review of the entire field of radio astronomy.

B. Jiang (Peking) and S. Truskin (Russian Academy of Sciences) have suggested improvements and also translated the fifth edition into Chinese and Russian, respectively.

1 Radio Astronomical Fundamentals

1.1 On the Role of Radio Astronomy in Astrophysics

Almost everything that we know about distant sources, that is, stars and the interstellar medium, has been obtained from electromagnetic radiation(自2016年宣布人类对于引力波的首个直接探测结果以来,现在190510还有引力波这一新的天

文研究窗口,引力波天文学). This includes **spatial distributions, kinematics and composition**. Only a very small part of our knowledge stems from material carriers, such as meteorites that impact the earth, cosmic ray particles or samples of material collected by manned or unmanned space probes.

For many thousands of years, mankind was restricted to measurements of visible light; only since the time of Herschel was this wavelength range slightly expanded into the near Infrared; in 1930, it extended from the near ultraviolet to the near infrared: 0.35 $\mu m \sim 1$ μm.

At other wavelengths, investigations were **limited either because the terrestrial atmosphere blocks radiation or because no detectors for this radiation were available**.

In 1931 this situation changed dramatically when Jansky showed that radiation at a wavelength of 14.6 m (= 20.5 MHz) received with a direction-sensitive antenna array followed siderial not solar time, so must be emitted by an extraterrestrial source which was not the sun.

Jansky continued his observations over several years without achieving much scientific impact:
Jansky, K. G. (1933a): Nature 13266:Radio Waves from Outside the Solar System
Jansky, K. G. (1933b): Proc. IRE 21 1387

BTW:
THE ORIGIN OF RADIO ASTRONOMY JANSKY, KARL G.

1932:Directional Studies of Atmospherics at High Frequencies.
这篇1932年的文章中,Jansky设计了一个可以输出信号源的方向的系统,有一个射电信号的方向以24小时为周期变化,Jansky当时很自然地以为它与太阳有关;一年后,随着观测数据的积累,他发现该信号其实以一年为周期移动,于是他想到这个源其实来自太阳系外的一个固定方向,这个结果1933年发在Nature:Jansky, K. G. (1933a),其实就是下面的这篇:
1933:Electrical Disturbances Apparently of Extraterrestrial Origin.

1935:A Note on the Source of Interstellar Interference.
这篇文章进一步总结了这几年的发现:任何时候只要天线系统指向银河系,都能探测到辐射信号,当指向银河系中心的时候,响应最强.进而得出这些辐射源位于这些银河系的恒星,或源自分布在银河系周围的星际介质.

1937:Minimum Noise Levels Obtained on Short-Wave Radio Receiving Systems.
这篇文章研究了环境噪声水平对射电信号探测的影响.It is pointed out that, on the shorter wave lengths and in the absence of man-made interference, the usable signal strength is generally limited by noise of interstellar origin. The powers obtained from this noise with the various antennas and for different times of the day are given.

1939:An Experimental Investigation of the Characteristics of Certain Types of Noise.
这篇文章研究了几种噪声(atmospheric noise and that due to the thermal agitation of electric charge in conductors; very sharp, widely separated, clean, noise pulses,)的带宽(band width)对有效电压、平均电压、峰值电压的影响.

The basic papers in the discovery of an entirely original and new branch of science, radio astronomy. This discovery is strictly comparable with Galileo's use of the telescope for astronomy, as it opened up a large, previously unsuspected field of investigation, and it very greatly increased the data available to astronomers.
Jansky's experiments were made at the Bell Telephone Laboratories in Holmdel, New Jersey, from 1931 to 1937.
In his first paper, he describes his finding of three sources of static noise, two from thunderstorms, and one from an unknown source, apparently extraterrestrial.
In the second and most important paper, he gives further information on the extraterrestrial noise, and suggests that it may originate in the Milky Way.
In his third paper, he locates the strongest source of the extra-terrestrial noise as the center of the Milky Way (the local galaxy).
In his fourth paper, he discusses interstellar noise as a limiting factor in the usefulness of radio systems, along with other interference.
Jansky died in 1950, several years before there was general appreciation of the great importance of his discovery.

Jansky's measurements were followed up and improved on by another radio engineer, Grote Reber, who carried out measurements at a shorter wavelength, $\lambda = 87$ m(=160 MHz). These observations were published in the Astrophysical Journal (Reber 1940:Notes: Cosmic Static.) Later, after the end of World War II, improved receivers allowed the new radio window to develop. Radio physics had made great progress during the war years, mainly due to the development of sensitive radar equipment. After the war, some researchers turned their attention towards the radio "noise" from extraterrestrial sources (Sullivan 1984:The Early Years of Radio Astronomy: Reflections Fifty Years After Jansky's Discovery, 2009:Sullivan, W.T. (2009): Cosmic Noise (Cambridge University Press, Cambridge)).

The historical development has been toward **higher sensitivity, shorter wavelength, and higher angular resolution**.
Today(2016), the radio astronomy window extends from $\lambda \sim 30$ m to $\lambda < 0.2$ mm.
This new astronomical discipline has been instrumental(起重要作用,the space program has always been instrumental in our efforts to make theoretical and technological advances) in changing our view of astronomy.
The results required emission mechanisms that differed considerably from those used previously. The objects studied in the optical wavelength range usually radiate because they are hot and therefore thermal physics is the rule. Often, in radio astronomy the radiation has a nonthermal origin and different physical mechanisms apply.

More recently, technological advances have opened up additional astronomical "windows".
Balloons, high-flying aircraft or satellites such as IRAS, ISO and MSX permitted observations in the mid and far infrared (FIR).
Other satellites such as IUE, Chandra, XMM-Newton and Fermi permitted measurements in the ultraviolet, X-ray and γ-ray wavelength ranges.
Rarer are satellites for the longest wavelengths, $\lambda > 30$m. Here, projects such as DARE, the Dark Ages Radio Explorer have been proposed.
Each of these spectral windows requires its own technology.
The art of carrying out measurements differs for each.
Astronomers have tended to view these different windows as forming different astronomies: radio astronomy, X-ray astronomy, infrared astronomy and so on.
Not only do wavelength range and (to some extent) technology differ.

The types of objects that emit at these wavelengths can also differ: some objects are detected only in certain spectral windows.
For example, diffuse cool gas in the form of atomic hydrogen is detected only because it emits or absorbs by means of the (first order forbidden) hyperfine structure line at $\lambda = 21$ cm; emission from this gas cannot be detected by any other means.
Similarly, this is true for denser, cool gas traced by the allowed rotational transitions of carbon monoxide, CO. This cooler material is detected only by molecular or atomic lines and broadband dust radiation.
Although interpretations differ for each spectral window there is one single reality.
An astrophysicist investigating a specific object collects information with optical, radio or other techniques. In this sense, today, there is no separate scientific discipline of radio astronomy.

New experimental techniques provide additional paths to attack old problems. More dramatically, when new kinds of objects are detected by these means, methods and results are organized to form a new discipline such as radio astronomy. However, **when the experimental methods have become mature and both the advantages and limitations of the methods become clearer, it is appropriate to integrate the specialized field into main stream astrophysics**. This is now the case for radio astronomy. The first, vigorous years when the pioneers worked alone or in small groups are over. Today radio astronomers rarely build complete systems, consisting of telescopes and receivers, by themselves.
This has profound effects on the way research is done. **In the pioneer days, a project usually started with a survey in which an instrument collected a large amount of data. In many cases the results were unusual and exciting, so these required new explanations. Now, more often, a researcher starts by posing a question and then searches for the means to answer it**.

Today, radio astronomy is a science which depends on complex instruments to gather the data, including the instrumental properties, advantages and limitations. These instruments are often no longer built by the user. Rather, **the task is to optimize the use of an instrument for a particular study. For this, the user must have a clear idea how the measurements are to be carried out**. Together with data reduction software, these are the hardware tools. **For the interpretation of the measurements, theoretical concepts must be applied to data. These concepts belong to a wide variety of physical fields, ranging from plasma to molecular physics. The applications of such concepts are tools**(Chap. 10).
Additional introductory texts on radio astronomy are by Kraus (1986), Burke and Graham-Smith (2009), Condon and Ransom (2007, an internet site), and Wielebinski and Klein (2010).

1.2 The Radio Window

This ground-based radio window extends roughly from a lower frequency limit of $\nu \sim 10$ MHz ($\lambda \sim 30$ m) to a high frequency cut-off at $\nu \sim 1.5$ THz ($\lambda \sim 0.2$ mm).

The **high-frequency cut-off occurs because the resonant absorption of the lowest rotation bands of molecules in the troposphere(对流层) fall into this frequency range**. Two molecules are mostly responsible for this: water vapor, H_2O and O_2.
Water vapor has many transitions, withe the most well-known having frequencies at $\nu = 22.2GHz(\lambda = 1.35cm)$ and 183 GHz (1.63 mm).

In the case of O_2, there is a group of lines or “band” at 60GHz (5 mm). Lines of O_2 consist of closely spaced levels of the ground electronic state, resulting in two interleaved series of absorption lines near 60 GHz (5 mm) and a single line near 119 GHz (2.52 mm).
The absorption of astronomical signals by other abundant molecules in the atmosphere, N_2 and CO_2, occurs at frequencies above 300 GHz.

There is great interest to extend the upper frequency limits of the measurements as high as possible, since astronomical sources produce more intense spectral lines in this range.
The rotational transitions of carbon monoxide, CO, play an especially important role since this molecule is very widespread and its chemistry is thought to be well understood.
The circumstance that **water vapor is one of the determining factors for this cut-off** makes it possible to extend the accessible frequency range somewhat by carrying out measurements from locations with a low total water vapor content. With respect to the absorption caused by oxygen little can be done from earth's surface. In some parts of the sub-mm wavelength range, measurements must be carried out from satellites such as the Herschel Space Observatory, the airborne facility SOFIA (Stratospheric Observatory for Infrared Astronomy), or from high flying balloons.
Interstellar spectral lines of water vapor and oxygen are best observed from satellites orbiting above the earth's atmosphere.
On earth, high altitude observatories with a dry climate are the best one can do.

At the lowest frequencies, the terrestrial atmosphere ceases to be transparent because of **free electrons in the ionosphere. Transmission through the atmosphere is not possible if the frequency of the radiation is below the plasma frequency** ν_p:

$$\frac{\nu_p}{kHz} = 8.97\sqrt{\frac{N_e}{cm^{-3}}},$$

where N_e is the electron density of the plasma. Thus the low-frequency limit or cutoff of the radio window will be near 4.5 MHz at night when the F_2 layer of the ionosphere has an maximum density, and near 11 MHz at daytime.
However the electron densities in the ionosphere depend on solar activity, and therefore this low-frequency limit varies with **“space weather”**. Only when the observing frequency is well above this limit do ionospheric properties have no noticeable effect. Radio astronomy in the range below the ionospheric cutoff must be performed from satellites above the earth's ionosphere.

Radio frequency interference (RFI,干扰,不是干涉) has an increasingly

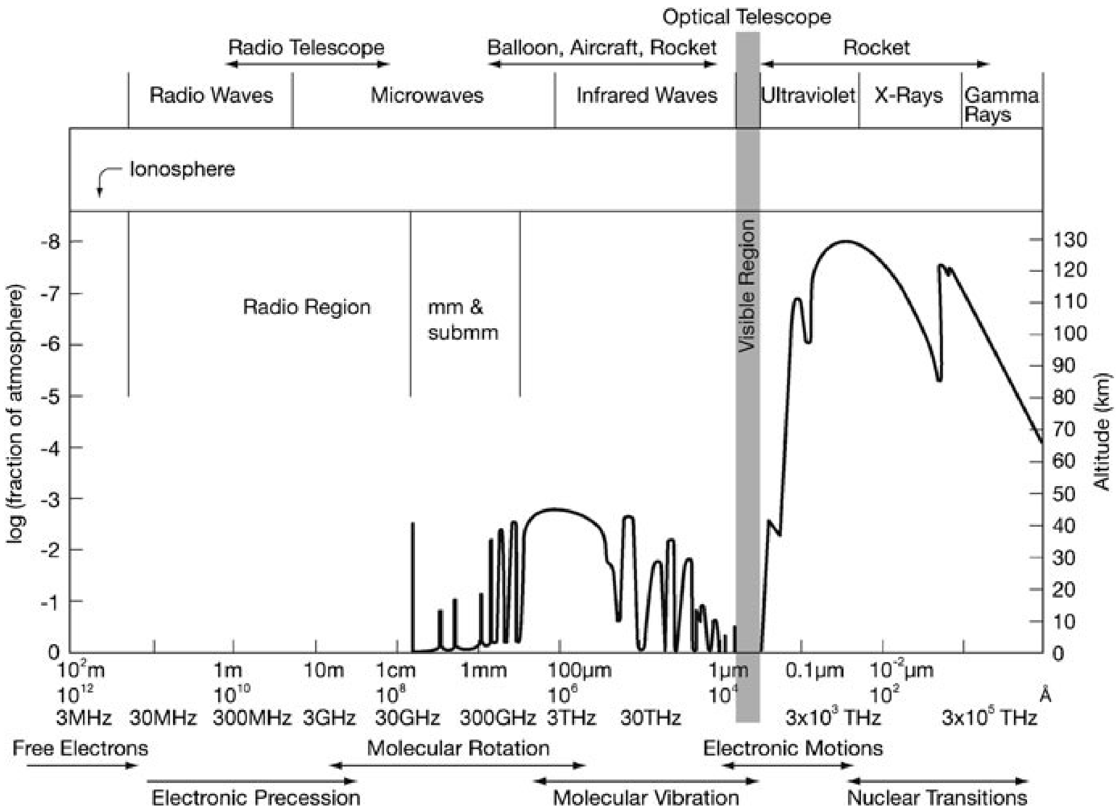

Figure 1: The transmission of the earth's atmosphere for electromagnetic radiation. The right hand vertical axis of this diagram gives the height in the atmosphere at which the radiation is attenuated by a factor 1/2

detrimental impact on astronomical observations.

Man-made sources of radio signals can completely dominate the very weak cosmic signals being studied.

Some forms of RFI can be partially removed, but the presence of RFI always compromises the utility of the data and/or the efficiency of data acquisition.

Information on the management of interference to radio astronomy can be found in the 1993: ITU Handbook on Radio Astronomy and 2005: CRAF Handbook for Radio Astronomy.

The Handbook has been prepared by the Committee on Radio Astronomy Frequencies of the European Science Foundation in Strasbourg, CRAF. It provides a comprehensive review of matters related to spectrum management and the protection of the science of Radio Astronomy against harmful interference. The review is placed within the historical and technological context within which the Radio

Astronomy Service operates.
This book is intended for a wide readership. It aims to provide a bridge between the professional radio astronomical community and professional radio spectrum managers with no previous background in astronomy.

1.3 Discoveries in Radio Astronomy

The history of radio astronomy is replete with major discoveries. The first was implicit in the data taken by Jansky, where the intensity of the extended radiation from the Milky Way exceeded that of the quiet Sun. This fact shows that radio and optical data must arise from different phenomena.
The radiation measured by Jansky was caused by the **synchrotron mechanism**; this interpretation was made more than 15 years later (see Rybicki and Lightman 1979: Radiative processes in astrophysics).
这是一本哈佛大学天文系教授写给研一学生的讲义,天体物理中的辐射过程.已经下载到Astronomy/Books文件夹.

The next discovery, in the 1940s, showed that the active Sun caused disturbances seen in radar receivers. In Australia, a unique instrument was used to associate this variable emission with sunspots (see Dulk 1985: Radio Emission from the Sun and Stars; Gary and Keller 2004:solar and space weather radio physics(已经下载到Astronomy/Books文件夹)).
Among the most important discoveries have been:

1. Discrete cosmic radio sources, at first, supernova remnants and radio galaxies (1948; see Kirshner 2004: The Extravagant Universe: Exploding Stars, Dark Energy, and the Accelerating Cosmos(已经下载到Astronomy/Books文件夹))

2. The 21cm line of atomic hydrogen (1951; see Sparke and Gallagher 2007: Galaxies in the Universe(已经下载到Astronomy/Books文件夹); Kalberla et al. , 2005:The Leiden/Argentine/Bonn (LAB) Survey of Galactic HI. Final data release of the combined LDS and IAR surveys with improved stray-radiation corrections)

3. Quasi Stellar Objects or "Quasars"(1963;see Begelman and Rees 2009: Gravity's Fatal Attraction: Black Holes in the Universe
(已经下载到Astronomy/Books文件夹))

4. The Cosmic Microwave Background (1965;see Silk 2008:The Infinite Cosmos: Questions from the Frontiers of Cosmology(已经下载到Astronomy/Books文件夹))

5. Interstellar molecules (1968;see Herbst and Dishoeck 2009: Complex Organic Interstellar Molecules) and the connection with Star Formation, including circumstellar and protoplanetary disks (see Stahler and Palla 2005: The Formation of Stars(已经下载到Astronomy/Books文件夹); Reipurth et al. 2007: PROTOSTARS AND PLANETS V(下载链接))

6. Pulsars(1968;see Lyne and Graham-Smith 2012:Pulsar Astronomy)

7. Distance determinations using source proper motions determined from Very Long Baseline Interferometry (see Reid 1993: The distance to the center of the Galaxy)

8. The Sunyaev-Zeldovich effect(see,e.g.Marrone et al.2012:LoCuSS: THE SUNYAEV–ZEL'DOVICH EFFECT AND WEAK-LENSING MASS SCALING RELATION)

9. Gravitational Lenses

10. Molecules in high redshift sources(see Solomon and VandenBout 2005: MOLECULAR GAS AT HIGH REDSHIFT)

These areas of research have led to investigations of the dynamics of galaxies, dark matter, general relativity, Black Holes, the early universe and gravitational radiation (see the overviews in Longair 2006:The Cosmic Century - A History of Astrophysics and Cosmology and Harwit 2006: The Cosmic Century - A History of Astrophysics and Cosmology(已下载)). In addition, follow ups of discoveries in other wavelength ranges have dramatically enlarged our knowledge of such sources. One example is the class of sources known as Ultra Luminous Infra Red Galaxies, or ULIRGs, first cataloged with the Infra Red Astronomy Satellite, IRAS. The closest example of a ULIRG is Arp 220, an energetic source, rich in molecules (see, e.g., Gonzales-Alfonso et al. 2013:HERSCHEL/SPIRE SUBMILLIMETER SPECTRA OF LOCAL ACTIVE GALAXIES; Konig et al. 2012:The Arp 220 merger on kpc scales).
Radio astronomy has been recognized by the physics community since four Nobel Prizes (1974, 1978, 1993, and 2006) were awarded for work in this field. In chemistry, the community has been made aware of the importance of a more general chemistry involving ions and molecules (see Herbst 2001: The chemistry of interstellar space). Two Nobel Prizes for chemistry have been awarded to persons engaged in molecular line astronomy.

1.3.1 A Selected List of Facilities

1.4 Some Basic Definitions

Electromagnetic radiation in the radio window is a wave phenomenon, but when the scale of the system involved is much larger than the wavelength, we can consider the radiation to travel in straight lines called **rays**.
Since the flux density of radio sources is usually very small, a special radio astronomical flux density unit, the Jansky (abbreviated Jy) has been introduced.
Rather few sources are as bright as 1 Jy, but even such a source would produce a signal of only 10^{15} W with the 100 m telescope (effective aperture A $\sim 5\times10^3 m^2$,$\Delta\nu =$ 20 MHz).

The brightness is independent of the distance.
The total flux density, S_ν, shows the expected dependence of $1/r^2$.
Another useful quantity related to the brightness is the radiation energy density μ_ν in units of erg cm^{-3}. From dimensional analysis, μ_ν is intensity divided by speed. Since radiation propagates with the velocity of light c, we have for the spectral energy density per solid angle:

$$\mu_\nu(\Omega) = \frac{1}{c} I_\nu$$

If integrated over the whole sphere, 4π steradian, results in the total spectral energy density μ_ν.

$$\mu_\nu = \frac{1}{c} \int_{(4\pi)} I_\nu d\Omega$$

1.5 Radiative Transfer

For radiation in free space the specific intensity I_ν remains independent of the distance along a ray.
I_ν will change only if radiation is absorbed or emitted, and this change of I_ν is described by the equation of transfer.

Equation of transfer:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

The linear absorption coefficient: κ_ν
Emissivity: ϵ_ν
There are several limiting cases for which the solution of the differential equation is especially simple.

1. Emission only: $\kappa_\nu = 0$
2. Absorption only: $\epsilon_\nu = 0$
3. Thermodynamic equilibrium (TE): If there is complete equilibrium of the radiation with its surroundings, the brightness distribution is described by the Planck function, which depends only on the thermodynamic temperature, T , of the surroundings

$$\frac{dI_\nu}{ds} = 0, I_\nu = B_\nu(T) = \epsilon_\nu / \kappa_\nu$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

4. Local thermodynamic equilibrium (LTE): Full thermodynamic equilibrium will be realized only in very special circumstances such as in a black enclosure or, say, in stellar interiors. Often Kirchhoff's law is:

$$\frac{\epsilon_\nu}{\kappa_\nu} = B_\nu(T)$$

applicable independent of the material, as is the case with complete thermodynamic equilibrium. In general, I_ν will differ from $B_\nu(T)$.

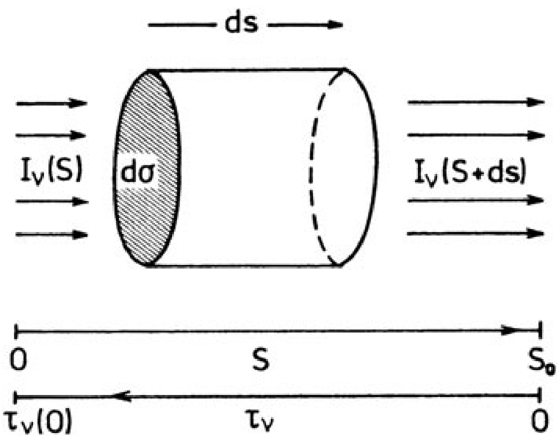

Figure 2: A sketch showing the quantities used in the equation of transfer

Define the **optical depth**: $d\tau_\nu$ by:

$$d\tau_\nu = -\kappa_\nu ds$$

接下来可以推导出:The observed brightness I_ν for the **optically thick** case is equal to the Planck blackbody brightness distribution independent of the material.

1.6 Black Body Radiation and the Brightness Temperature

The spectral distribution of the radiation of a black body in thermodynamic equilibrium is given by the Planck law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

It gives the power per unit frequency interval.
积分→The total brightness of a black body is obtained:

$$B(T) = \sigma T^4, \sigma = ...$$

Above equation is the **Stefan-Boltzmann radiation law** which was found experimentally in 1879 by J. Stefan and derived theoretically in 1884 by L. Boltzmann before Planck's radiation law was known.

$B_\nu(T)$ has a maxima: **Wien's displacement law**

$$(\frac{\lambda_{max}}{cm})(\frac{T}{K}) = 0.28978$$

后面介绍了在实际中各个公式的应用场景:(原文P16,书的P32) This can be used for all thermal radio sources except perhaps for low tempera- tures in the millimeter or sub-mm range. In practice, Eq. (1.28) is used for studies of cosmic sources at centimeter and longer wavelengths. An important exception is the study of planets; for this, the use of the Planck temperature is standard. Studies of the 2.73 K microwave background must use Planck temperatures (see Fig. 1.7). For receiver calibrations using the y factor (see Eq. 4.34, the use of Eq. 1.28 is the norm).

One of the important features of the Rayleigh-Jeans law is **the implication that the brightness and the thermodynamic temperature of the black body that emits this radiation are strictly proportional (1.28). This feature is so useful that it has become the custom in radio astronomy to measure the brightness of an extended source by its brightness temperature T_B.** This is the temperature which would result in the given brightness if inserted into the Rayleigh-Jeans law

$$T_B = \frac{c^2}{2k}\frac{1}{\nu^2}I_\nu = \frac{\lambda^2}{2k}I_\nu$$

然后结果简单推导,就能得出(书P33):

$$(\frac{S_\nu}{Jy}) = 2.65T_B(\frac{\theta}{arcminutes})^2(\frac{\lambda}{cm})^{-2}$$

That is, **with a measurement of the flux density S in Janskys, and the source size, the brightness temperature, T_B, of the source can be determined in the Rayleigh-Jeans approximation**.
It should be mentioned here that for studies of planets, the Planck temperature is the one normally used. If emitted by a black body and $h \ll kT$ then T_B gives the thermodynamic temperature of the source, a value that is independent of. If other processes are responsible for the emission of the radiation, Tb will depend on the frequency; it is, however, still a useful quantity and is commonly used in practical work.
推导可得出光学厚和光学薄时,T(s)与T_b的关系:
T(s) is the thermodynamic temperature of the medium at the position s
T_b: radiation temperature/brightness temperature

- For optically thin $\tau \ll 1$:
$$T_b = \tau_\nu T$$
- For optically thick $\tau \gg 1$:
$$T_b = T$$

1.7 Emissivity and Reflectivities of Surfaces

The discussion so far is concerned with Black Body temperatures. For most sources, this is an idealization. **Black Bodies are perfect absorbers, and, in equilibrium, perfect emitters. In nature perfect absorbers or emitters are very rare**. Thus, there must be a factor to take this into account. This is the **reflectivity**, r. This factor is defined as:

$$T_{b_0} = (1 - r)T_0 + rT_s$$

where T_0 is the temperature of a source, and Ts is the temperature of an intervening medium. Clearly if r is unity, the medium is opaque and the source is undetectable. In the other limit, $r = 0$, and the medium is transparent. This concept has a variety of applications......

The surface of a reflector: the reflection from a highly polished surface is has the value $r \sim 1$. Thus the temperature of the reflecting surface plays only a small role in the sensitivity in the radio range. This is **Specular Reflection**镜面反射. This is certainly the case of antenna surfaces at longer wavelengths, but not in the infrared.

1.8 The Nyquist Theorem and the Noise Temperature

Finally, we relate electrical power and temperature.
......
In thermal equilibrium, this power is determined by the physical temperature.
......

$$P_\nu = kT$$

Then, the available noise power of a resistor is proportional to its temperature, the noise temperature T_N, and independent of the value of R. Throughout the whole radio range, from the longest waves to the far infrared region the noise spectrum is white, that is, its power is independent of frequency. **For receivers, since the impedance of a noise source must be matched to that of the amplifier, such a noise source can only be matched over some finite bandwidth.(这句话不理解)**

Not all circuit elements can be characterized by thermal noise. For example a

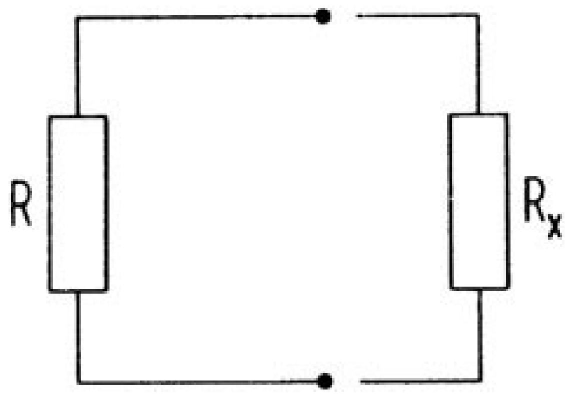

Figure 3: A sketch of a circuit containing a resistor R, to illustrate the origin of Johnson noise. The resistor R, on the left, at a temperature T , provides a power kT to a matched load R_X , on the right

microwave oscillator can deliver the equivalent of more than 10^{16} K, although the physical temperature is only 300 K. Clearly this is a very nonthermal process, and in this case temperature is not a useful concept.

Problems

1. If the average electron density in the interstellar medium (ISM) is $0.03 cm^{-3}$, what is the lowest frequency of electromagnetic radiation which one can receive due to the effect of this plasma? Compare this to the ionospheric plasma

cutoff frequency if the electron density, Ne, in the ionosphere is $\sim 10^5 cm^{-3}$.

Use the equation on page 4 of 'Tools', which is: $\nu_p = 8.97 \times (0.03)^{0.5}$.

At the lowest frequencies, the terrestrial atmosphere ceases to be transparent because of **free electrons in the ionosphere. Transmission through the atmosphere is not possible if the frequency of the radiation is below the plasma frequency** ν_p:

$$\frac{\nu_p}{kHz} = 8.97\sqrt{\frac{N_e}{cm^{-3}}},$$

where N_e is the electron density of the plasma. Thus the low-frequency limit or cutoff of the radio window will be near 4.5 MHz at night when the F_2 layer of the ionosphere has an maximum density, and near 11 MHz at daytime.

Solution:
The equation given in the statement of this problem determines the cut-off frequency of a plasma. For the Interstellar Medium (ISM), that is, $\nu_p = 8.97 \times (0.03)^{0.5} = 1.6kHz$, while the ionospheric cutoff is $\nu_p = 8.97 \times (10^5)^{0.5} = 2.8MHz$, from the discussion on page 4 of 'Tools'. **Since the plasma cut-off frequency in the ISM is much lower than the cutoff frequency in the Earth's ionosphere, there may be astronomical phenomena which can be observed only from above the ionosphere. Thus such measurements must be made from satellites**.

2. (a) A researcher measures radio emission at a frequency of 250 kHz and finds that the emission is present over the whole sky with a brightness temperature of 250 K. Could the origin of this radiation be the earth's ionosphere?
Solution: From the result of problem 1, this radiation must arise in the ionosphere.

(b) Assume that the source fills the entire visible sky, taken to be a half hemisphere. What is the power received by an antenna with A = 1 m^2 collecting area in a B = 1 kHz bandwidth?
Solution: Assuming that this is black body radiation which falls on the antenna from one hemisphere (solid angle $\Omega = 2\pi$ steradians), we use the Rayleigh–Jeans law to relate the brightness temperature to the power. The antenna can receive only one polarization, so the relation for power, P , is $P = S_\nu \times Area \times Bandwidth \times SolidAngle$ and $S_n u = 2kT/\lambda^2 \times SolidAngle$: $P = (2kT/\lambda^2)AB(1/2)2\pi$. For these values, the resulting power is 1.5×10^{-23} W.

PS:steradians is the SI unit of solid angle, equal to the angle at the center of a sphere subtended by a part of the surface equal in area to the square of the radius:$\frac{Surface_{area}}{r^2}$
origin: late 19th century: from Greek stereos 'solid'+ radian.

3. The downward-pointing radar satellite, Cloudsat, is moving in a polar orbit at an altitude of 500 km. The operating frequency is 94 GHz. Assume that the power is radiated over a hemisphere. The peak power will be 1500 W, uniformly distributed over a bandwidth of 1 GHz. If no power is absorbed in the earth's atmosphere, what is the peak flux density of this satellite when it is directly overhead? This radar is transmitting 3% of the time (duty cycle). What is the average power radiated and the corresponding flux density?

   Solution: The peak flux density when it is directly overhead is:
   $S = 1500W/[2\pi(5 \times 10^5 m)^2 \times (10^9 Hz)] = 9.6 \times 10^{-19} Wm^{-2}Hz^{-1} = 9.6 \times 10^7 Jy$. The average power is 3% of this value or 2.9×10^6 Jy.

   94 GHz 没有用到.

4. A unit commonly used in astronomy is flux density, S_ν , the Jansky (Jy). One Jy is $10^{-26} Wm^{-2}Hz^{-1}$. Calculate the flux density, in Jy, of a microwave oven with an output of 1 kW at a distance of 10 m if the power is radiated over all angles and is uniformly emitted over a bandwidth of 1 MHz.

   Solution: Inserting the values give $S_\nu = 600W/[4\pi(10m)^2(10^6 Hz)] = 4.8 \times 10^{-7} Wm^{-2}Hz^{-1} = 4.8 \times 10^{19} Jy$.

5. (a) What is the flux density, S_ν , of a source which radiates a power of 1 kW in the microwave frequency band uniformly from 2.6 to 2.9 GHz, when placed at the distance of the Moon ($3.84 \times 10^5 km$)? Repeat for an identical source if the radiation is in the optical frequency band, from 3×10^{14} to 8×10^{14} Hz.

   Solution: In the microwave band, the flux is $S_\nu = 1000W/[4\pi(3.84 \times 10^8 m)^2 \times (3 \times 10^8 Hz)] = 1.8 \times 10^{-24} Wm^{-2}Hz^{-1} = 1.8 \times 10^2 Jy$. In the optical range, the bandwidth is much larger ($5 \times 10^{14} Hz$), resulting in a far smaller value, $S_\nu = 5 \times 10^{-4} Jy$.

   (b) If we assume that the number of photons is uniform over the band, what is the average energy, $E = h\nu$, of a photon? Use this average photon energy and the power to determine N , the number of photons. How many photons pass through a $1m^2$ area in one second in the optical and radio frequency bands?

Solution: In the microwave range, the average photon energy is $E_r(photon) = h\nu = (6.62 \times 10^{-27} ergs) \times (2.8 \times 10^9 Hz) = 1.85 \times 10^{-17} erg = 1.85 \times 10^{-24} J$. In the optical, it is $E_o(photon) = 3.6 \times 10^{-12} erg = 3.6 \times 10^{-19} J$. The total number of photons emitted per second is $N_r = 5.49 \times 10^{27}$ (radio) and $N_o = 3.0 \times 10^{21}$ (optical).
The photon flux can be calculated using N(photon)/A, where A is the area of the sphere with the radius being the distance to the Moon. The photon fluxes are $3.0 \times 10^8 m^{-2}s^{-1}$(radio) and $1620m^{-2}s^{-1}$ (optical), respectively.

6. In the near future there may be an anti-collision radar installed on automobiles. It will operate at 78.5 GHz. If the bandwidth is 10 MHz, and at a distance of 3 m, the power per area is $10^{-9}Wm^{-2}$. Assume the power level is uniform over the entire bandwidth of 10 MHz. What is the flux density of this radar at 1 km distance? A typical large radio telescope can measure to the mJy ($= 10^{-29}Wm^{-2}Hz^{-1}$) level. At what distance will such radars disturb such radio astronomy measurements?

   Solution: At 1 km, $S = 10^{-9}Wm^{-2}(3/1000)^2/(10^8 Hz)] = 9 \times 10^{-23}Wm^{-2}Hz^{-1}$ The result is $9 \times 10^3 Jy$. If there is a line of sight to the telescope, there will be interference at a level of at least the sensitivity limit of the telescope up to a distance of 3000 km, if there is no atmospheric absorption. (这后面的没懂) The actual distance will be smaller if the radiation follows a straight line path. For this distance, D, if the height is small compared to the radius of the Earth, Re, we have

   $$D = \sqrt{2hR_e}$$

   For a height, h, of 1 km, the range of the interference would be limited to 80 km.

7. If the intensity of the Sun peaks in the optical range, at a frequency of about 3.4×10^{14} Hz, what is the temperature of the Sun? Use the Wien displacement law. If all of the power is emitted only between 3 and 4×10^{14} Hz, how many photons per cm^2 arrive at the earth when the Sun is directly overhead? What is the power received on earth per cm^2? A value for the solar power is 135 mW per cm^2. How does this compare to your calculation?

   $$\frac{v_{\max}}{GHz} = 58.789 \frac{T}{K}$$

   We have $v_{\max} = 3.4 \times 10^{14}\text{Hz} = 3.4 \times 10^5\text{GHz} = 58.789\text{T}$, thus $T = 5780K$.

The number of photons per second is n. In the (simple but rather unrealistic) monoenergetic case, we use $n = L_{\odot}/hv = 1.7 \times 10^{45}$ photons emitted per second. At $1AU = 1.5 \times 10^{13}$cm, this number is reduced by a factor of $4\pi\left(1.5 \times 10^{13}\right)^2 = 2.8 \times 10^{27} cm^2$, so the number received on earth is $6.0 \times 10^{17} cm^{-2} s^{-1}$. Multiplying by the energy of each photon, $2.25 \times 10^{-12} erg$, we have $1.4 \times 10^6 ergcm^{-2}s^{-1} = 0.14 Wcm^{-2} = 1.4 kWm^{-2} = 1.4 mWcm^{-2}$. So, good agreement.

Problems number 8 and 9 were shifted to a later chapter.

8. shifted to a later chapter

9. shifted to a later chapter

10. Show that (1.34) can be obtained from (1.33). Extend the relation to arcseconds, wavelength in millimeters and milli Jy, to obtain:

$$\left(\frac{\mathrm{S}_v}{\mathrm{mJy}}\right) = 73.6\mathrm{T}_\mathrm{B}\left(\frac{\theta}{\text{arc seconds}}\right)^2\left(\frac{\lambda}{\mathrm{mm}}\right)^{-2}$$

Eq. (1.34):

$$\left(\frac{S_v}{\mathrm{Jy}}\right) = 2.65\mathrm{T}_\mathrm{B}\left(\frac{\theta}{\mathrm{arcmin}}\right)^2\left(\frac{\lambda}{\mathrm{cm}}\right)^{-2}$$

From the Rayleigh-Jeans relation in MKS units, we have from Eq. (1.33):

$$\mathrm{S}_v = 2\mathrm{kT}_\mathrm{B}\theta^2/\lambda^2\Delta\Omega$$

11. Suppose the maximum observed temperature of the galaxy is 10^5 K at 14.6 m wavelength. If this is measured with a 24° gaussian beam, what is the flux density? If the bandwidth used is 100kHz and the collecting area is $800m^2$, what is the received power?
Use the result of problem 10 or Eq. (1.34):

$$S_v(\mathrm{Jy}) = 2.65 \times 10^5 \times (24 \times 60)^2/\left(14.6 \times 10^2\right)^2 = 2.58 \times 10^5 \mathrm{Jy}$$

For the power, P, we multiply S_ν by the collecting Area, A and bandwidth, B. The result is $P = 2.58 \times 10^5 \times 10^3 Hz \times 800m^2$. Then the power is: $P = 2.1 \times 10^{-13} W$.

2 Electromagnetic Wave Propagation Fundamentals

2.1 Maxwell's Equations

2.2 Plane Waves in Nonconducting Media

2.3 Wave Packets and the Group Velocity

2.4 Plane Waves in Conducting Media

2.5 The Dispersion Measure of a Tenuous Plasma

dispersion measure:

$$\mathrm{DM} = \int_0^L \left(\frac{N}{\mathrm{cm}^{-3}}\right) \mathrm{d}\left(\frac{l}{\mathrm{pc}}\right)$$

$$\frac{\Delta\tau_{\mathrm{D}}}{\mu\mathrm{s}} = 4.148 \times 10^9 \left[\frac{\mathrm{DM}}{\mathrm{cm}^{-3}\mathrm{pc}}\right] \left[\frac{1}{\left(\frac{v_1}{\mathrm{MHz}}\right)^2} - \frac{1}{\left(\frac{v_2}{\mathrm{MHz}}\right)^2}\right]$$

Since both the time delay $\Delta\tau_{\mathrm{D}}$ and the observing frequencies v_1 and v_2 can be measured with high precision, a very accurate value of DM for a given pulsar can be determined from

$$\frac{\mathrm{DM}}{\mathrm{cm}^{-3}\mathrm{pc}} = 2.410 \times 10^{-4} \left(\frac{\Delta\tau_{\mathrm{D}}}{\mathrm{s}}\right) \left[\frac{1}{\left(\frac{v_1}{\mathrm{MHz}}\right)^2} - \frac{1}{\left(\frac{v_2}{\mathrm{MHz}}\right)^2}\right]^{-1}$$

Provided the distance L to the pulsar is known, this gives a good estimate of the average electron density between observer and pulsar. However since L is usually known only very approximately, only approximate values for N can be obtained in this way. Quite often the opposite procedure is used: From reasonable guesses for N , a measured DM provides information on the unknown distance L to the pulsar.

Dispersion in the ISM, combined with a finite pulse width alimit to the fine structure one can resolve in a pulse. The frequency dependence of the pulse arrival time is τ_{from} (2.67). This gives a condition for the bandwidth b needed to resolve a feature in a time τ .

$$\left(\frac{b}{\mathrm{MHz}}\right) = 1.205 \times 10^{-4} \frac{1}{\left[\frac{\mathrm{DM}}{\mathrm{cm}^{-3}\mathrm{pc}}\right]} \left[\frac{v}{\mathrm{MHz}}\right]^3 \left[\frac{\tau}{\mathrm{s}}\right]$$

since the pulses will have a finite width in both time and frequency, a differential form of (2.73) will give a limit to the maximum bandwidth that can be used at a given frequency and DM if a time resolution τ is wanted. This will be rediscussed in the context of pulsar back ends.

2.6 Problems

3 Wave Polarization

3.1 Vector Waves

3.2 The Poincaré Sphere and the Stokes Parameters

Poincaré introduced another representation that permits an easy visualization of all the different states of polarization of a vector wave.
If we interpret the angles 2ψ of (3.19) and 2χ of (3.21) as longitude and latitude on a sphere with the radius S_0 of (3.15) there is a one-to-one relation between polarization states and points on the sphere (Fig. 3.2). The equator represents linear polarization; the north pole corresponds to right-circular and the south pole to left-circular polarization (Fig. 3.3).

Finally, one should note that so far we have implied (but not explicitly stated) that a strictly monochromatic wave is always polarized; there is no such thing as an unpolarized monochromatic wave.
This situation will be different when we consider quasi-monochromatic radiation, in which ω is restricted to some small but finite bandwidth. Radiation of this kind can be unpolarized or partially polarized. To analyze this, one must have a convenient way to describe such radiation. This will be done in the next section.

3.3 Quasi-monochromatic Plane Waves

$$\left\langle A^2(t)\right\rangle = \langle VV^*\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} V(t)V^*(t)\mathrm{d}t$$

If we require that $\langle A^2\rangle$ has a finite value, then $\int_{-\infty}^{\infty} VV^*$ diverges, However, according to Wiener (1949), the techniques of Fourier analysis can be extended to such a generalized harmonic analysis; therefore we will assume that time averaged values for A can be computed from (3.47).

3.4 The Stokes Parameters for Quasi-monochromatic Waves

$$\begin{aligned}
S_0 &= I = \langle a_1^2 \rangle + \langle a_2^2 \rangle \\
S_1 &= Q = \langle a_1^2 \rangle - \langle a_2^2 \rangle \\
S_2 &= U = 2 \langle a_1 a_2 \cos\delta \rangle \\
S_3 &= V = 2 \langle a_1 a_2 \sin\delta \rangle
\end{aligned}$$

and these can be calculated from six intensity measurements. Using (3.51) we find

$$\begin{aligned}
S_0 &= I = I\,(0^\circ, 0) + I\,(90^\circ, 0) \\
S_1 &= Q = I\,(0^\circ, 0) - I\,(90^\circ, 0) \\
S_2 &= U = I\,(45^\circ, 0) - I\,(135^\circ, 0) \\
S_3 &= V = I\left(45^\circ, \tfrac{\pi}{2}\right) - I\left(135^\circ, \tfrac{\pi}{2}\right)
\end{aligned}$$

For partially polarized light we find from

$$\begin{aligned}
S_0^2 &\geqq S_1^2 + S_2^2 + S_3^3 \\
I^2 &\geqq Q^2 + U^2 + V^2
\end{aligned}$$

instead of (3.30), which is valid for strictly monochromatic waves. It is then easy to express **the degree of polarization**

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^3}}{S_0}$$

The Stokes parameters of the superposition of several independent vector waves will be the sum of the Stokes parameters of the individual waves.

3.5 Faraday Rotation

In 1845, Faraday detected that the polarization angle of dielectric materials will rotate if a magnetic field is applied to the material in the direction of the light propagation. This indicated to him that light must be an electromagnetic phenomenon. In radio astronomy this Faraday rotation has become an important tool to investigate the interstellar magnetic field (see, e.g., Figure. 4). As shown in Sect. 2.5 interstellar gas must be treated as a tenuous plasma. Wave propagation in such a medium in the presence of an external magnetic field is a rather complicated subject with many different wave modes, cut-offs, etc. It is treated rather extensively in most textbooks on plasma physics and we refer to a few of these in the reference list for this chapter.

In Sect. 2.4, we have obtained the dispersion equation linking wave number $k = 2\pi/\lambda$, and circular frequency $\omega = 2\pi v$ for wave propagation in a dispersive medium. In Sect. 2.5, we studied wave propagation in a tenuous plasma by examining the effects of the conductivity σ on an electromagnetic wave in a medium with free electrons. Here we will repeat this process but will include an external magnetic field.

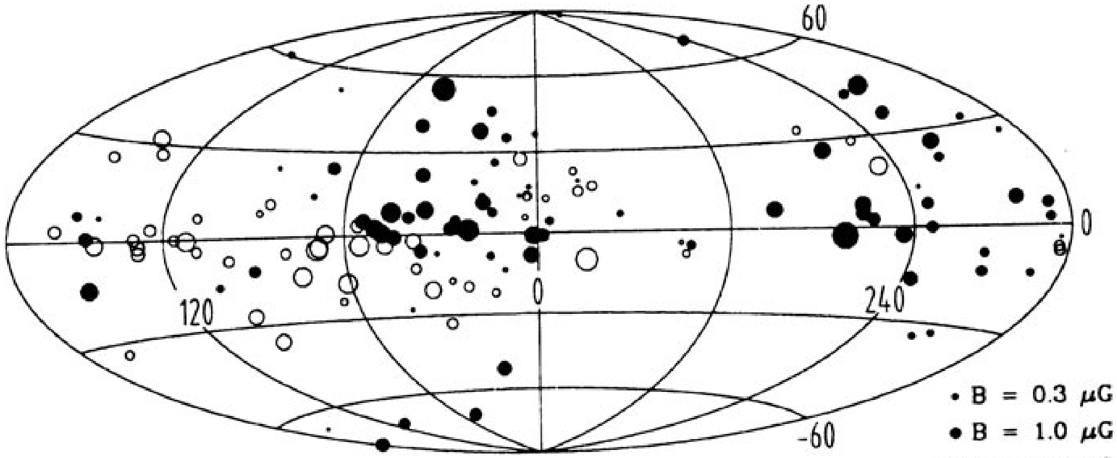

Figure 4: A plot of the line-of-sight magnetic field strength determined from Faraday rotation. From the rotation measure and dispersion measure one can obtain the column density of electrons. This data is for pulsars with distances <3 kpc. Positive fields are shown by filled circles, negative fields by open circles. The size of the symbols are proportional to field strength (adapted from Backer, in Verschuur and Kellermann, 1988)

For a finite slab with variable density $N(z)$ and magnetic flux density $B(z)$, we thus obtain the total rotation of the polarization direction

$$\Delta\psi = \frac{e^3}{2\pi m^2 c}\frac{1}{v^2}\int_0^L B_{\|}(z)N(z)\mathrm{d}z$$

In astronomy a system of mixed units is usually employed. Using this system, we have

$$\frac{\Delta\psi}{\mathrm{rad}} = 8.1\times 10^5 \left(\frac{\lambda}{\mathrm{m}}\right)^2 \int_0^{L/\mathrm{pc}} \left(\frac{B_{\|}}{\mathrm{Gauss}}\right)\left(\frac{N_e}{\mathrm{cm}^{-3}}\right)\mathrm{d}\left(\frac{z}{\mathrm{pc}}\right)$$

The dependence of $\Delta\psi$ on v^{-2} can be used to determine the value of $\int BN\ \mathrm{d}\ z$ from the measurement of the polarization direction at two frequencies:

$$\begin{aligned}\frac{\mathrm{RM}}{\mathrm{radm}^{-2}} &= 8.1\times 10^5 \int \left(\frac{B_{\|}}{\mathrm{Gauss}}\right)\left(\frac{N_e}{\mathrm{cm}^{-3}}\right)\mathrm{d}\left(\frac{z}{\mathrm{pc}}\right)\\ &= \frac{\left(\frac{\Delta\psi_1}{\mathrm{rad}}\right)-\left(\frac{\Delta\psi_2}{\mathrm{rad}}\right)}{\left(\frac{\lambda_1}{\mathrm{m}}\right)^2-\left(\frac{\lambda_2}{\mathrm{m}}\right)^2}\end{aligned}$$

In this expression the unknown intrinsic polarization angle of the source cancels. The units of RM are radians per m^2, and positive RM indicates that $B_{\|}$ points toward us. Equation (3.71) can, conversely, be used to determine the intrinsic polarization angle from (3.70) and thus be used to correct the measured polarization. For pulsars, one can combine the values of RM from the Faraday rotation of pulsars and DM, from the pulse dispersion from (2.69). The resulting ratio gives the average magnetic field parallel to the line-of-sight

$$\left(\frac{\overline{B}_{\|}}{\text{Gauss}}\right) = 1.23 \times 10^{-6} \left(\frac{RM}{DM}\right)$$

If there are line-of-sight reversals, $B_{\|}$ is a lower limit to the actual value. Results for pulsars at distances less than 3 kpc show a scatter, but in the galactic longitude range $00^{\circ} - 180^{\circ}$, the direction of $\overline{B}_{\|}$ is away from the Sun, and at longitudes $180^{\circ} - 360^{\circ}$, towards the Sun. This is in the sense of galactic rotation. The $\overline{B}_{\|}$ fields obtained from pulsar studies are in the range of $0.3 - 3\mu$ Gauss. Faraday rotation measurements in our galaxy can be affected by field reversals. This is especially the case for the inner parts of our galaxy, where reversals in B field direction are thought to be present.

3.6 Problems

4 Signal Processing and Receivers: Theory

In this chapter, topics concerned with signal processing and noise analysis are covered in Sect. 4.1. These lead to the fundamental results in Sect. 4.2 and Appendix C which are needed to understand the properties of radiometers. Specifics of actual receivers will be presented in the next chapter. The topics presented in this Chapter are essential for an understanding of the concepts presented in Chaps.5–9. A working knowledge of Fourier transforms is needed for all of the material presented. We give a summary of the relevant concepts of Fourier transforms (FT), including convolutions and related topics in Appendix A.

4.1 Signal Processing and Stationary Stochastic Processes

4.1.1 Probability Density, Expectation Values and Ergodicity

the expectation value $E\{f(x)\}$ of a function $f(x)$ is given by

$$E\{f(x)\} = \int_{-\infty}^{\infty} f(x)p(x)\mathrm{d}x$$

This is different from the expected value of the transformation $y = f(x)$

$$E\{y\} = \int_{-\infty}^{\infty} y p_y(y)\mathrm{d}y = \int_{-\infty}^{\infty} f(x)p_x(x)\frac{\mathrm{d}x}{|f'(x)|}$$

4.1.2 Autocorrelation and Power Spectrum

Therefore the simple definition for the FT

$$X(v) = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\mathrm{e}^{-2\pi \mathrm{i} v t}\mathrm{d}t$$

does not exist; the integral varies irregularly as T increases. As first shown by N. Wiener, the concept of the Cesaro sum of an improper integral can be used to advantage in this situation. The Cesaro sum is defined as

$$\int_{-\infty}^{\infty} A(x)\mathrm{d}x = \lim_{N\to\infty} \frac{1}{N} \int_0^N \left[\int_{-r}^{r} A(x)\mathrm{d}x\right] \mathrm{d}r$$

that is, as the limit of the average over the finite integrals. This limit will exist for a wide class of functions where the ordinary improper integral does not exist.

Because $x(t)$ is assumed to be stationary, we must have
$R_T(\tau) = E_T\{x(s)x(s+\tau)\} = E_T\{x(t-\tau)x(t)\}$

where $R_T(\tau)$ is the autocorrelation function (ACF). Introducing the ACF into the above expression and performing the integration with respect to s, we find

$$E_T\left\{|X(v)|^2\right\} = T\int_{-T}^{T}\left(1-\frac{|\tau|}{T}\right) R_T(\tau)\mathrm{e}^{-2\pi\mathrm{i}v\tau}\mathrm{d}\tau$$

But the right-hand side is a Cesaro sum, and therefore by defining the power spectral density (PSD), $S(v)$, as

$$S(v) = \lim_{T\to\infty} \frac{1}{T} E_T\left\{|X(v)|^2\right\}$$

we obtain from $E_T\left\{|X(v)|^2 =\right\} = ...$

$$S(v) = \int_{-\infty}^{\infty} R(\tau)\mathrm{e}^{-2\pi\mathrm{i}v\tau}\mathrm{d}\tau$$

This is the Wiener-Khinchin theorem stating that the ACF,$R(\tau)$, and the PSD,$S(\nu)$, of an ergodic random process are FT pairs (see a graphical representation in Figure. 5). Taking the inverse FT of (4.14) we obtain

$$R(\tau) = \int_{-\infty}^{\infty} S(v)\mathrm{e}^{2\pi\mathrm{i}v\tau}\mathrm{d}v$$

Thus the total power transmitted by the process is given by

$$R(0) = \int_{-\infty}^{\infty} S(v)\mathrm{d}v = E\left\{x^2(t)\right\}$$

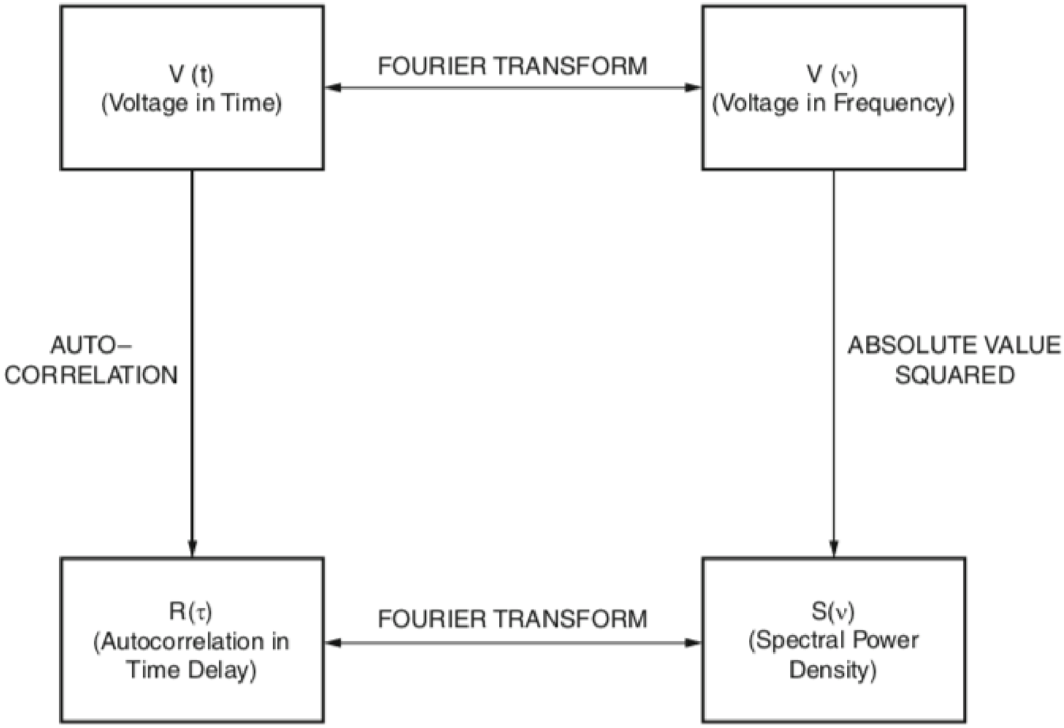

Figure 5: A sketch of the relation between the voltage input as a function of time, $V(t)$, and frequency,$V(\nu)$, with the autocorrelation function, ACF, $R(\tau)$, where is the delay, and corresponding power spectral density, PSD, $S(\nu)$. The two-headed arrows represent reversible processes

The limit $T \to \infty$ of the autocorrelation function (ACF)$R_T(\tau)$ can be found using the Cesaro sum resulting in

$$R(\tau) = E\{x(s)x(s+\tau)\} = \lim_{T\to\infty} \int_{-T}^{T} \left(1 - \frac{|s|}{2T}\right) x(s)x(s+\tau)\mathrm{d}s$$

Using the concept of ensemble average, this can also be written as

$$R(\tau) = \int\int_{-\infty}^{\infty} x_1(s)x_2(s+\tau)p\left(x_1, x_2; \tau\right)\mathrm{d}x_1\mathrm{d}x_2$$

where $p\left(x_1, x_2; \tau\right)$ is the joint probability density function for the appearance of values x_1 and x_2 which are separated by the time τ. For ergodic stationary processes, (4.17) and (4.18) lead to identical results, but sometimes one or the other is easier to apply.

4.1.3 Linear Systems

4.1.4 Filters

Filters are devices that limit the frequencies passed through a system or change the phase of an input.

4.1.5 Digitization and Sampling

The essential part of any digital system is the device that produces a digital output from the analog input. Functionally, the operation of such devices can be divided into two parts:

1. Analog-to-Digital converters (A/D converters) and
2. Samplers.

4.1.6 Gaussian Random Variables

4.1.7 Square Law Detectors

$$\sigma_y^2 = E\left\{y^2(t)\right\} - E^2\{y(t)\} = 2E^2\{y(t)\}$$

Thus, in contrast to linear systems, the mean value of the output signal of a square-law detector does not equal zero, even if the input signal has a zero expected mean value.

4.1.8 Receiver Calibration Procedure

There are two types of receivers: direct detector systems and heterodyne systems. In radio astronomy the noise power of coherent receivers (those which preserve the phase of the input) is usually measured in terms of the noise temperature. To calibrate a receiver, one relates the noise temperature increment ΔT at the receiver input to a given measured receiver output increment,Δz provided the detector characteristics are known. In practice the receiver is calibrated by connecting two or more known power sources to the input.

4.2 Limiting Receiver Sensitivity

Receivers are devices used to measure the PSD. Both direct detector and heterodyne systems contain the following basic units:

1. A reception (usually band pass) filter that defines the spectral range of the receiver.

2. A square-law detector used to produce an output signal that is proportional to the average power in the reception band.

3. A smoothing filter or averager, which determines the time response of the output.

In some cases, processes in (2) and (3) are carried out after digitizing the signal, so the operations could be carried out in a computer. In some cases, a receiver might record the sampled voltages on a storage device for later analysis (see, e.g., Problem 1(c) for example). In other cases, for a fast receiver response, item (3) might be dispensed with.
A receiver must be able to detect faint signals in the presence of noise, that is, be sensitive. Just as with any other measuring device there are limits for this sensitivity, since the receiver input and the receiver itself are affected by noise.
Even when no input source is connected to a receiver, there is an output signal, since any receiver generates thermal noise. This noise is amplified together with the signal. Since signal and noise have the same statistical properties, these cannot be distinguished.
Next, receiver noise properties will be related to the minimum uncertainty in a measure- ment.

4.2.1 Noise Uncertainties Due to Random Processes

$$\frac{\Delta T}{T_{\rm sys}} = \frac{1}{\sqrt{\Delta \upsilon \tau}}$$

Equation (4.42) is the fundamental relation between system noise, bandwidth, integration time and rms fluctuations: For a given system, the improvement in the RMS noise cannot be better than as given in Eq. (4.42). Systematic errors will only increase ΔT, although the time behavior may follow relation (4.42). We repeat for emphasis: T_{sys} is the noise from the entire system. That is, it includes the noise from the receiver, atmosphere, ground, and the source. Therefore ΔT will be larger for an intense source. However this is the ideal situation since the receiver noise is dominated by the signal. This is the case for measurements of galactic radiation at frequencies below a few hundred MHz. For direct detection systems, usually Bolometers, the system noise is given in terms of Noise Equivalent Power, NEP. The definition of NEP can be related to system noise; see Sect. 5.1.4 for further details.

4.2.2 The Minimum Noise for a Coherent System

Coherent receivers are usually heterodyne receivers. These preserve phase, so are subject to a condition that gives rise to a theoretical minimum value for the receiver noise. Incoherent receivers do not preserve phase, so have no theoretical minimum receiver noise. The following development provides an important limit to the noise temperature for coherent receivers. This limit for a coherent receiver or phase-preserving amplifier is obtained by application of the Heisenberg uncertainty principle.

$$T_{\mathrm{rx}}(\text{ minimum }) = \frac{hv}{k}$$

In the centimeter and even millimeter wavelength regions, this noise temperature limit is quite small. For example, at 2.6mm , it is 5.5K . However, at a wavelength of 0.3mm, the limit is 47.8K . Presently, the best receiver noise temperatures are $\cong$ 5 times these values. This derivation is valid for receiver noise temperatures

that are rather far above the quantum limits. As pointed by Kerr et al. (1996), for receiver noise temperatures below 40K, an additional effect, the zero point energy,

is important. Expressed in temperature units, this is $\frac{hv}{2k}$. In practice, this effect will raise the receiver noise estimate by $\approx 10\%$.

4.2.3 Receiver Stability

$$\frac{\Delta T_{\mathrm{RMS}}}{T_{\mathrm{R}} + T_{\mathrm{A}}} = \frac{\Delta G}{G}$$

This shows that variations of the output power caused by gain variations enter directly into the determination of limiting sensitivity.
This was first applied to radio astronomical receivers by Dicke (1946). This is a straightforward application of the compensation principle such as the Wheatstone bridge.

$$\frac{\Delta T_{\mathrm{RMS}}}{T_{\mathrm{R}}} = \frac{\Delta G}{G} \frac{\mathrm{T_A} - T_{\mathrm{ref}}}{T_{\mathrm{R}}}$$

The influence of gain fluctuations depends on the difference $\mathrm{T_A} - T_{\mathrm{ref}}$

In the millimeter and sub-mm wavelength range, the sky temperature can have large fluctuations and this have a large effect on TA. Then compensation involves a determination of sky conditions by measuring a nearby region of sky at the receiver frequency. This is referred to as "beam switching". A well-known application of beam switching is the "Differential Microwave Radiometer" DMR instrument on-board the "Cosmic Microwave Background Explorer" satellite, COBE. With such a process, a measurement accuracy is much better than 10^{-4}.

At all wavelengths, fluctuations in the atmosphere will affect high resolution images. At millimeter and sub-mm wavelengths, these fluctuations are mostly due to water vapor, but for wavelengths in the range of a meter, the ionospheric fluctuations can distort images. Corrections for such effects are complex, and will be described in Chap. 9.

1. Effect of Switching on Receiver Noise
   The time spent measuring the reference using Dicke switching will not contribute to an improvement in the S/N ratio
   At frequencies below a few hundred MHz, source noise dominates, so the total power output of a single dish may be sufficient. Total power measurement was the procedure used by Jansky and Reber.

   If the time variation of G is included in the expression for the sensitivity limit, the generalization of (4.42) for stochastic time variations of $\Delta G/G$ will be Eq. (4.56).

$$\frac{\Delta T}{T_{\rm sys}} = K\sqrt{\frac{1}{\Delta \upsilon \tau} + \left(\frac{\Delta G}{G}\right)^2}$$

   The Allan plot is the ultimate way to measure stability, but requires a great amount of measurement time. Therefore it is often used to test receivers in a laboratory, but only rarely on telescopes.

4.3 Problems

5 Practical Receiver Systems

5.1 Historical Introduction

5.1.1 Incoherent Radiometers

Incoherent radiometers do not preserve phase; these operate as direct detection systems. The most common type of incoherent radiometers is the bolometer, used at millimeter wavelengths. Bolometers are basically very sensitive thermometers, so have no frequency or polarization specific response, wide bandwidths and are sensitive to both polarizations. For single telescope continuum measurements in the millimeter and sub-mm ranges, semiconductor bolometers have dominated the field because of their high sensitivities. In the past few years, superconducting bolometers have come into use (see, e.g. Rieke 2002). For a history, see Low et al. (2007).

5.1.2 Coherent Radiometers

5.1.3 Bolometer Radiometers

The operation of a bolometer makes use of the effect that the resistance, R, of a material varies with the temperature. When radiation is absorbed by the bolometer material, the temperature varies; this temperature change is a measure of the intensity of the incident radiation. Because this thermal effect is rather independent of the frequency of the radiation absorbed, bolometers are intrinsically broadband devices. The frequency discrimination needed must be provided by external filters. A bias voltage must be applied to a bolometer for optimum performance. Although of great practical importance, especially for superconducting bolometers, we neglect the bias voltage in the following. This treatment follows the analysis of Mather (1982) and Jones (1953).

5.1.4 The Noise Equivalent Power of a Bolometer

A frequently used measure for the quality of a detector is its noise equivalent power, NEP, is the power which must fall on the detector to raise output by an amount equal to the RMS noise of the detector. The NEP is measured as the response to a sinusoidally modulated input which is switched between two temperatures. The units of NEP are Watts $Hz^{-1/2}$.

5.1.5 Currently Used Bolometer Systems

1. Superconducting Bolometers
2. Polarization Measurements
3. Spectral Line Measurements

5.2 Coherent Receivers

5.2.1 Basic Components: Passive Devices

1. Thermal Noise of an Attenuator
2. Isolators
3. Directional Couplers

5.2.2 Basic Components: Active Devices

- Phase Lock Systems
- Amplifier
- Mixers
- Local Oscillator Sources

5.2.3 Semiconductor Junctions