An Introduction to Modern Astrophysics
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Reading Notes

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Contents

Preface

The progresses in Astrophysics during the past decade.

It was just two months before the printing of the first editions that Michel Mayor and Didier Queloz announced the discovery of an extrasolar planet around 51 Pegasi, the first planet found orbiting a main-sequence star.
These discoveries shed new light on how stars and planetary systems form, inform us about formation and planetary evolution in our own Solar System.

2003 UB313 challenging our definition of what a planet is and how many planets our Solar System is home to

The armada of orbiters, and rovers: Spirit and Opportunity, have confirmed that liquid water has existed on the surface of Mars in the past. Robotic emissaries visit Jupiter and Saturn, touch down on the surfaces of Titan and asteroids, crash into cometary nuclei, and even return cometary dust to Earth

Swift $\rightarrowtail$ gamma-ray bursts, one class of gamma-ray bursts is associated with core-collapse supernovae and the other class is probably associated with the merger of two neutron stars, or a neutron star and a black hole, in a binary system

Remarkably precise observations of the center of our Milky Way Galaxy and other galaxies, revealed that a great many, perhaps most, spiral and large elliptical galaxies are home to one or more supermassive black holes at their centers. Galactic mergers help to grow these monsters in their centers. Supermassive black holes are the central engines responsible for the exotic and remarkably energetic phenomena associated with radio galaxies, Seyfert galaxies, blazars, and quasars

The expansion of the universe is not slowing down but accelerating! $\rightarrow$ We currently live in a dark-energy-dominated universe, in which Einstein's cosmological constant (once considered his "greatest blunder") plays an important role in our understanding of cosmology. Dark energy was not even imagined in cosmological models at the time the first editions were published.

Cosmology has entered into a new era of precision measurements.
Wilkinson Microwave Anisotropy Probe (WMAP) $\rightarrow$ previously large uncertainties in the age of the universe have been reduced to less than 2% (13.7 ± 0.2 Gyr). Meanwhile, stellar evolution theory and observations $\rightarrow$ the determination that the ages of the oldest globular clusters are in full agreement with the upper limit of the age of the universe.

We opened the preface to the first editions with the sentence "There has never been a more exciting time to study modern astrophysics";
Joining the Hubble Space Telescope in its high-resolution study of the heavens have been the Chandra X-ray Observatory and the Spitzer Infrared Space Telescope. From the ground, 8-m and larger telescopes have also joined the search for new information about our remarkable universe. Tremendously ambitious sky surveys have generated a previously unimagined wealth of data that provide critically important statistical data sets; the Sloan Digital Sky Survey, the Two-Micron All Sky Survey, the 2dF redshift survey, the Hubble Deep Fields and Ultradeep Fields, and others have become indispensable tools for hosts of studies. We also anticipate the first observations from new observatories and spacecraft, including the high-altitude (5000 m) Atacama Large Millimeter Array and high-precision astrometric missions such as Gaia and SIM PlanetQuest. Of course, studies of our own Solar System also continue; just the day before this preface was written, the Mars Reconnaissance Orbiter entered orbit around the red planet

We are confident that BOB and its smaller siblings will serve the needs of a range of introductory astrophysics courses and that they will instill some of the excitement felt by the authors and hosts of astronomers and astrophysicists worldwide

We have switched from cgs to SI units in the second edition.

Our goal in writing these texts was to open the entire field of modern astrophysics to you by using only the basic tools of physics. Nothing is more satisfying than appreciating the drama of the universe through an understanding of its underlying physical principles. The advantages of a mathematical approach to understanding the heavenly spectacle were obvious to Plato, as manifested in his Epinomis:
Are you unaware that the true astronomer must be a person of great wisdom? Hence there will be a need for several sciences. The first and most important is that which treats of pure numbers. To those who pursue their studies in the proper way, all geometric constructions, all systems of numbers, all duly consti-

tuted melodic progressions, the single ordered scheme of all celestial revolutions should disclose themselves. And, believe me, no one will ever behold that spectacle without the studies we have described, and so be able to boast that they have won it by an easy route.
Now, 24 centuries later, the application of a little physics and mathematics still leads to deep insights.

A course in astrophysics offers students the unique opportunity of actually using the physics they have learned to appreciate many of astronomy's fascinating phenomena. Furthermore, as a discipline, astrophysics draws on virtually every aspect of physics. Thus astrophysics gives students the chance to review and extend their knowledge.

Anyone who has had an introductory calculus-based physics course is ready to understand nearly all the major concepts of modern astrophysics. The amount of modern physics covered in such a course varies widely, so we have included a chapter on the theory of special relativity and one on quantum physics which will provide the necessary background in these areas. Everything else in the text is self-contained and generously cross-referenced, so you will not lose sight of the chain of reasoning that leads to some of the most astounding ideas in all of science.

Although we have attempted to be fairly rigorous, we have tended to favor the sort of back-of-the-envelope calculation that uses a simple model of the system being studied. The payoff-to-effort ratio is so high, yielding 80% of the understanding for 20% of the effort, that these quick calculations should be a part of every astrophysicist's toolkit. In fact, while writing this book we were constantly surprised by the number of phenomena that could be described in this way. Above all, we have tried to be honest with you; we remained determined not to simplify the material beyond recognition. Stellar interiors, stellar atmospheres, general relativity, and cosmology — all are described with a depth that is more satisfying than mere hand-waving description

Computational astrophysics is today as fundamental to the advance in our understanding of astronomy as observation and traditional theory, and so we have developed numerous computer problems, as well as several complete codes, that are integrated with the text material. You can calculate your own planetary orbits, compute observed features of binary star systems, make your own models of stars, and reproduce the gravitational interactions between galaxies. These codes favor simplicity over sophistication for pedagogical reasons; you can easily expand on the conceptually transparent codes that we have provided. Astrophysicists have traditionally led the way in large-scale computation and visualization, and we have tried to provide a gentle introduction to this blend of science and art

An extensive website at http://www.aw-bc.com/astrophysics is associated with these texts. It contains downloadable versions of the computer codes in various languages, including Fortran, C++, and, in some cases, Java. There are also links to some of the many important websites in astronomy. In addition, links are provided to public domain images found in the texts, as well as to line art that can be used for instructor presentations. Instructors may also obtain a detailed solutions manual directly from the publisher

Chapter 1

The Celestial Sphere

1.1 THE GREEK TRADITION

Human beings have long looked up at the sky and pondered its mysteries
The modern discipline of astronomy depends heavily on a mathematical formulation of its physical theories, following the process begun by the ancient Greeks.

1.1.1 The Geocentric Universe

The stars of the night sky revolved about a fixed Earth and that the heavens ought to obey the purest possible form of motion. Plato therefore proposed that celestial bodies should move about Earth with a uniform (or constant) speed and follow a circular motion with Earth at the center of that motion. This concept of a geocentric universe was a natural consequence of the apparently unchanging relationship of the stars to one another in fixed constellations

1.1.2 Retrograde Motion

Attempting to understand this backward, or retrograde, motion became the principal problem in astronomy for nearly 2000 years!

Despite its shortcomings, the Ptolemaic model became almost universally accepted as the correct explanation of the motion of the wandering stars. When a disagreement between the model and observations would develop, the model was modified slightly by the addition of another circle. This process of “fixing” the existing theory led to an increasingly complex theoretical description of observable phenomena.

一个离真相很 远的理论模型, 即使暂时能解释观测数据, 但随着时间的演变, 观测与理论模型的偏差将越来越大. 就算在该理论框架下做微调, 也只能暂时解

决当前的偏差, 而为了与观测吻合, 理论模型本身会变得十分复杂. 就像 Epicycle 理论, 到后期本轮和均轮加起来很多个了, 而真实图景: 日心而非地心, 万有引力主导下的开普勒运动在数学和物理上都是极为简单优美的. 不是说 Epicycle 理论不好, 人类对周围世界的认识本来就是循序渐进的, 任何做出探索和思考的人都是值得敬佩的. 我们可以把这样的探索历史作为我们现在探索的一个参考, 提示我们如何去接近真相, 鉴别真相. Actually, Aristarchus proposed a heliocentric model of the universe in 280 b.c. At that time, however, there was no compelling evidence to suggest that Earth itself was in motion.(广泛而细致地读一读大师们的作品, 有些思想和理论在当时和当下看会有不同的效果, 当时无法验证的思想, 现在能不能想办法验证?)

1.2 THE COPERNICAN REVOLUTION

Polishborn astronomer Nicolaus Copernicus (1473–1543), hoping to return the science to a less cumbersome, more elegant view of the universe, suggested a heliocentric (Sun-centered) model of planetary motion. His bold proposal led immediately to a much less complicated description of the relationships between the planets and the stars.

1.2.1 Bringing Order to the Planets

The fact that Mercury and Venus are never seen more than 28° and 47°, respectively, east or west of the Sun clearly establishes that their orbits are located inside the orbit of Earth.
The Copernican model also predicts that only inferior planets can pass in front of the solar disk (inferior conjunction), as observed.

1.2.2 Retrograde Motion Revisited

The great long-standing problem of astronomy—retrograde motion—was also easily explained through the Copernican model.(一个困扰人类很久有些"古怪"的观测现象, 可能被一个形式简单优美的理论来解释)

会合周期 S 的计算思路: 内部行星比外部行星多绕 2π, 为下一次会合.

Although the Copernican model did represent a simpler, more elegant model of planetary motion, it was not successful in predicting positions any more accurately than the Ptolemaic model. This lack of improvement was due to Copernicus's inability to relinquish the 2000-year-old concept that planetary motion required circles, the human notion of perfection.(根深蒂固的观念有时会阻碍认识的进步,

这个宇宙在最根本的一些规律主导下, 自由演化, 所以轨道很难是完美的圆. 人生的发展轨迹亦是如此, 在性格/习惯/环境/理想等的主导下, 每个人都在自我修炼和与他人交互的过程中演化, 很难是个完美的轨迹, 但至少这轨迹大部分可以由自己的理想主导演化) As a consequence, Copernicus was forced (as were the Greeks) to introduce the concept of epicycles to "fix" his model.

Perhaps the quintessential example of a scientific revolution was the revolution begun by Copernicus. What we think of today as the obvious solution to the problem of planetary motion—a heliocentric universe—was perceived as a very strange and even rebellious notion during a time of major upheaval.

Thomas Kuhn has suggested that an established scientific theory is much more than just a framework for guiding the study of natural phenomena. The present paradigm (or prevailing scientific theory) is actually a way of seeing the universe around us. We ask questions, pose new research problems, and interpret the results of experiments and observations in the context of the paradigm. Viewing the universe in any other way requires a complete shift from the current paradigm. To suggest that Earth actually orbits the Sun instead of believing that the Sun inexorably rises and sets about a fixed Earth is to argue for a change in the very structure of the universe, a structure that was believed to be correct and beyond question for nearly 2000 years. Not until the complexity of the old Ptolemaic scheme became too unwieldy could the intellectual environment reach a point where the concept of a heliocentric universe was even possible.

1.3 POSITIONS ON THE CELESTIAL SPHERE

1.3.1 The Altitude–Azimuth Coordinate System

Although simple to define, the altitude–azimuth system is difficult to use in practice. Coordinates of celestial objects in this system are specific to the local latitude and longitude of the observer and are difficult to transform to other locations on Earth. Also, since Earth is rotating, stars appear to move constantly across the sky, meaning that the coordinates of each object are constantly changing, even for the local observer. Complicating the problem still further, the stars rise approximately 4 minutes earlier on each successive night, so that even when viewed from the same location at a specified time, the coordinates change from day to day

1.3.2 Daily and Seasonal Changes in the Sky

1.3.3 The Equatorial Coordinate System

1.3.4 Precession

Precession is the slow wobble of Earth's rotation axis due to our planet's non-spherical shape and its gravitational interaction with the Sun and the Moon.

1.3.5 Measurements of Time

Because of the need to measure events very precisely in astronomy, various high-precision time measurements are used. For instance, Heliocentric Julian Date (HJD) is the Julian Date of an event as measured from the center of the Sun. In order to determine the heliocentric Julian date, astronomers must consider the time it would take light to travel from a celestial object to the center of the Sun rather than to Earth. Terrestrial Time (TT) is time measured on the surface of Earth, taking into consideration the effects of special and general relativity as Earth moves around the Sun and rotates on its own axis

1.3.6 Archaeoastronomy

1.3.7 The Effects of Motions Through the Heavens

Consider the velocity of a star relative to an observer (Fig. 15). The velocity vector may be decomposed into two mutually perpendicular components, one lying along the line of sight and the other perpendicular to it. The line-of-sight component is the star's **radial velocity**, $\mathbf{v_r}$; the second component is the star's **transverse or tangential velocity**, $\mathbf{v}_\theta$, along the celestial sphere. This transverse velocity appears as a slow, angular change in its equatorial coordinates, known as **proper motion**.

1.3.8 An Application of Spherical Trigonometry

Definition of **position angle**.

1.4 PHYSICS AND ASTRONOMY

The mathematical view of nature first proposed by Pythagoras and the Greeks led ultimately to the Copernican revolution. The inability of astronomers to accurately fit the observed positions of the "wandering stars" with mathematical models resulted in a dramatic change in our perception of Earth's location in the universe. However, an equally important step still remained in the development of science: the search for physical causes of observable phenomena. As we will see the modern study of astronomy relies heavily on an understanding of the physical nature of the universe. The application of physics to astronomy, astrophysics, has proved very successful in explaining a wide range of observations, including strange and exotic objects and events, such as pulsating stars, supernovae, variable X-ray sources, black holes, quasars, gamma-ray bursts, and the Big Bang.

As a part of our investigation of the science of astronomy, it will be necessary to study the details of celestial motions, the nature of light, the structure of the atom, and the shape of space itself. Rapid advances in astronomy over the past several decades have occurred because of advances in our understanding of fundamental physics and because of improvements in the tools we use to study the heavens: telescopes and computers.

Essentially every area of physics plays an important role in some aspect of astronomy. Particle physics and astrophysics merge in the study of the Big Bang; the basic question of the origin of the zoo of elementary particles, as well as the very nature of the fundamental forces, is intimately linked to how the universe was formed. Nuclear physics provides information about the types of reactions that are possible in the interiors of stars, and atomic physics describes how individual atoms interact with one another and with light, processes that are basic to a great many astrophysical phenomena. Condensed-matter physics plays a role in the crusts of neutron stars and in the center of Jupiter. Thermodynamics is involved everywhere from the Big Bang to the interiors of stars. Even electronics plays an important role in the development of new detectors capable of giving a clearer view of the universe around us.

With the advent of modern technology and the space age, telescopes have been built to study the heavens with ever-increasing sensitivity. No longer limited to detecting visible light, telescopes are now capable of "seeing" gamma rays, X-rays, ultraviolet light, infrared radiation, and radio signals. Many of these telescopes require operation above Earth's atmosphere to carry out their missions. Other types of telescopes, very different in nature, detect elementary particles instead of light and are often placed below ground to study the heavens

Computers have provided us with the power to carry out the enormous number of calculations necessary to build mathematical models from fundamental physical principles. The birth of high-speed computing machines has enabled astronomers to calculate the evolution of a star and compare those calculations with observations; it is also possible to study the rotation of a galaxy and its interaction with neighboring galaxies. Processes that require billions of years (significantly longer than any National Science Foundation grant) cannot possibly be observed directly but may be investigated using the modern supercomputer.

All of these tools and related disciplines are used to look at the heavens with a probing eye. The study of astronomy is a natural extension of human curiosity in its purest form. Just as a small child is always asking why this or that is the way it is, the goal of an astronomer is to attempt to understand the nature of the universe in all of its complexity, simply for the sake of understanding—the ultimate end of any intellectual adventure. In a very real sense, the true beauty of the heavens lies not only in observing the stars on a dark night but also in considering the delicate interplay between the physical processes that cause the stars to exist at all

The most incomprehensible thing about the universe is that it is comprehensible. — Albert Einstein

The Celestial Sphere - PROBLEM SET

1. Derive the relationship between a planet's synodic period and its sidereal period.
   在相邻两次回合期间, 内行星比外行星多转 2π:

$$2\pi = \frac{2\pi}{P_{in}}S - \frac{2\pi}{P_{out}}S$$

   so,

$$\frac{1}{S} = \frac{1}{P_{in}} - \frac{1}{P_{out}}$$

Chapter 2

Celestial Mechanics

2.1 ELLIPTICAL ORBITS

2.1.1 Tycho Brahe: The Great Naked-Eye Observer

Despite the great care with which he carried out his work, Tycho was not able to find any clear evidence of the motion of Earth through the heavens, and he therefore concluded that the Copernican model must be false.

2.1.2 Kepler's Laws of Planetary Motion

In particular, the discrepant points were each off by approximately $8'$, or twice the accuracy of Tycho's data. Believing that Tycho would not have made observational errors of this magnitude, Kepler felt forced to dismiss the idea of purely circular motion (观测数据精确的重要性)
Kepler's three laws of planetary motion.
Kepler's third law was published ten years later(重大的科学发现需要时间的积淀, 尽管10年对每个人来说挺长, 但也只是历史长河中的一小部分) In retrospect it is easy to understand why the assumption of uniform and circular motion first proposed nearly 2000 years earlier was not determined to be wrong much sooner; in most cases, planetary motion differs little from purely circular motion. In fact, it was actually fortuitous that Kepler chose to focus on Mars, since the data for that planet were particularly good and Mars deviates from circular motion more than most of the others.(科学需要精确, 如果不是第谷的足够精确的观测, 也很难有新的发现; 科学也需要"运气", 但只要不想当然, 即使开普勒研究别的行星, 发现它们的轨道与圆偏差不大, 只要他接着研究下一颗行星, 迟早也会发现火星轨道的"不同")

2.1.3 The Geometry of Elliptical Motion

2.2 NEWTONIAN MECHANICS

2.2.1 The Observations of Galileo

2.2.2 Newton's Three Laws of Motion

Isaac Newton (1642–1727), arguably the greatest of any scientific mind in history, was born on Christmas Day in the year of Galileo's death.
In the two years following the completion of his formal studies, and while living at home in Woolsthorpe, in rural England, away from the immediate dangers of the Plague, Newton engaged in what was likely the most productive period of scientific work ever carried out by one individual.

Concerning the successes of his own career, Newton wrote:
I do not know what I may appear to the world; but to myself I seem to have been only like a boy, playing on the seashore, and diverting myself, in now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me.

2.2.3 Newton's Law of Universal Gravitation

从开普勒第三定律和牛顿的运动定律(圆周运动), 可以推导出万有引力定律.

The shell acts gravitationally as if its mass were located entirely at its center.

2.2.4 The Orbit of the Moon

2.2.5 Work and Energy

In astrophysics, as in any area of physics, it is often very helpful to have some understanding of the energetics of specific physical phenomena in order to determine whether these processes are important in certain systems. Some models may be ruled out immediately if they are incapable of producing the amount of energy observed. Energy arguments also often result in simpler solutions to particular problems.

2.3 KEPLER'S LAWS DERIVED

万有引力定律的发现又为开普勒定律提供了物理上的解释.
Although Kepler did finally determine that the geometry of planetary motion was in the more general form of an ellipse rather than circular motion, he was unable to explain the nature of the force that kept the planets moving in their precise patterns. Not only was Newton successful in quantifying that force, he was also able to generalize Kepler's work, deriving the empirical laws of planetary motion from the gravitational force law. The derivation of Kepler's laws represented a crucial step in the development of modern astrophysics.

2.3.1 The Center-of-Mass Reference Frame

Definition of the **reduced mass**

2.3.2 The Derivation of Kepler's First Law

The angular momentum of a system is a constant for a central force law
. The path of the reduced mass about the center of mass under the influence of gravity (or any other inverse-square force) is a conic section.

2.3.3 The Derivation of Kepler's Second Law

The total energy of a binary orbit depends only on the semimajor axis a and is exactly one-half the time-averaged potential energy of the system

$$E = \frac{1}{2} \langle U \rangle$$

where $\langle U \rangle$ denotes an average over one orbital period.5 This is one example of the virial theorem, a general property of gravitationally bound systems.

2.3.4 The Derivation of Kepler's Third Law

Not only did Newton demonstrate the relationship between the semimajor axis of an elliptical orbit and the orbital period, he also found a term not discovered empirically by Kepler, the square of the orbital period is inversely proportional to the total mass of the system. (Because the mass of the sun is much larger than planets in our solar system.)

The importance to astronomy of Newton's form of Kepler's third law cannot be overstated.(This sentence can be a template to show A's importance to B)

2.4 THE VIRIAL THEOREM

The total energy of the binary orbit was just one-half of the time-averaged gravitational potential energy or $E = \langle U \rangle /2$. Since the total energy of the system is negative, the system is necessarily bound. For gravitationally bound systems in equilibrium, it can be shown that the total energy is always one-half of the time-averaged potential energy; this is known as the **virial theorem**.

The virial theorem applies to a wide variety of systems, from an ideal gas to a cluster of galaxies. For instance, consider the case of a static star. In equilibrium a star must obey the virial theorem, implying that its total energy is negative, one-half of the total potential energy. Assuming that the star formed as a result of the gravitational collapse of a large cloud (a nebula), the potential energy of the system must have changed from an initial value of nearly zero to its negative static value. This implies that the star must have lost energy in the process, meaning that gravitational energy must have been radiated into space during the collapse.

Celestial Mechanics - PROBLEM SET - COMPUTER PROBLEMS

Chapter 3

The Continuous Spectrum of Light

3.1 STELLAR PARALLAX

Measuring the intrinsic brightness of stars is inextricably linked with determining their distances

Finding the distance to astronomical objects is one of the most important and most difficult tasks faced by astronomers.

The true scale of the Solar System was first revealed in 1761 when the distance to Venus was measured as it crossed the disk of the Sun in a rare transit during inferior conjunction. The method used was trigonometric parallax, the familiar surveyor's technique of triangulation. Similarly, the distances to the planets can be measured from two widely separated observation sites on Earth.

A star may also change its position as a consequence of its own motion through space. However, this proper motion, seen from Earth, is not periodic and so can be distinguished from the star's periodic displacement caused by Earth's orbital motion.

By definition, when the parallax angle $p = 1''$, the distance to the star is **1 pc**, thus 1 parsec is the distance from which the radius of Earth's orbit, 1AU, subtends an angle of $1''$

In fact, this cyclic change in a star's position is so difficult to detect that it was not until 1838 that it was first measured, by Friedrich Wilhelm Bessel (1784–1846), a

German mathematician and astronomer
In 1838, after 4 years of observing 61 Cygni, Bessel announced his measurement of a parallax angle of $0.316'' \sim 3.16$ pc阿 for that star, within 10% of the modern value 3.48 pc.

Tycho Brahe had searched for stellar parallax 250 years earlier, but his instruments were too imprecise to find it. Tycho concluded that Earth does not move through space, and he was thus unable to accept Copernicus's model of a heliocentric Solar System(由于第谷数据的精确, 开普勒发现了太阳系内行星运行的规律, 由于第谷的数据不够精确, 所以没有发现背景恒星的视差, 也就仍然认为地球静止不动.)

Stellar trigonometric parallax is currently useful only for surveying the local neighborhood of the Sun.

In addition, ESA will launch the Gaia mission within the next decade as well, which will catalog the brightest 1 billion stars with parallax angles as small as 10 microarcseconds.
Clearly these ambitious projects will provide an amazing wealth of new information about the three-dimensional structure of our Galaxy and the nature of its constituents.

3.2 THE MAGNITUDE SCALE

Nearly all of the information astronomers have received about the universe beyond our Solar System has come from the careful study of the light emitted by stars, galaxies, and interstellar clouds of gas and dust. Our modern understanding of the universe has been made possible by the quantitative measurement of the intensity and polarization of light in every part of the electromagnetic spectrum

3.2.1 Apparent Magnitude

In the nineteenth century, it was thought that the human eye responded to the difference in the logarithms of the brightness of two luminous objects.
The magnitudes discussed in this section are actually **bolometric magnitudes**, measured over all wavelengths of light; see Section 6 for a discussion of magnitudes measured by detectors over a finite wavelength region

inverse square law for light

3.2.2 Absolute Magnitude

3.2.3 The Distance Modulus

The quantity $m - M$ is therefore a measure of the distance to a star and is called the star's distance modulus:

$$m - M = 5log_{10}\frac{d}{10pc}$$

At first glance, it may seem that astronomers must start with the measurable quantities F and m and then use the distance to the star (if known) to determine the star's intrinsic properties. However, if the star belongs to an important class of objects known as pulsating variable stars, its intrinsic luminosity L and absolute magnitude M can be determined without any knowledge of its distance. Equation (5) then gives the distance to the variable star.

3.3 THE WAVE NATURE OF LIGHT

Much of the history of physics is concerned with the evolution of our ideas about the nature of light.

3.3.1 The Speed of Light

3.3.2 Young's Double-Slit Experiment

3.3.3 Maxwell's Electromagnetic Wave Theory

Ten years after Maxwell's death, the German physicist Heinrich Hertz (1857–1894) succeeded in producing radio waves in his laboratory. Hertz determined that these electromagnetic waves do indeed travel at the speed of light, and he confirmed their reflection, refraction, and polarization properties. In 1889, Hertz wrote: *What is light? Since the time of Young and Fresnel we know that it is wave motion. We know the velocity of the waves, we know their lengths, and we know that they are transverse; in short, our knowledge of the geometrical conditions of the motion is complete. A doubt about these things is no longer possible; a refutation of these views is inconceivable to the physicist. The wave theory of light is, from the point of view of human beings, certainty.*

3.3.4 The Electromagnetic Spectrum

Today, astronomers utilize light from every part of the electromagnetic spectrum.

3.3.5 The Poynting Vector and Radiation Pressure

Like all waves, electromagnetic waves carry both energy and momentum in the direction of propagation. The rate at which energy is carried by a light wave is described by the **Poynting vector**

Radiation pressure has a negligible effect on physical systems under everyday conditions. However, radiation pressure may play a dominant role in determining some aspects of the behavior of extremely luminous objects such as early main-sequence stars, red supergiants, and accreting compact stars. It may also have a significant effect on the small particles of dust found throughout the interstellar medium.

3.4 BLACKBODY RADIATION

3.4.1 The Connection between Color and Temperature

Because an ideal emitter reflects no light, it is known as a **blackbody**, and the radiation it emits is called **blackbody radiation**. Stars and planets are blackbodies, at least to a rough first approximation.

Wien's displacement law

3.4.2 The Stefan–Boltzmann Equation

Josef Stefan (1835–1893) in 1879 showed that the luminosity, L, of a blackbody of area A and temperature T (in kelvins) is given by

$$L = A\sigma T^4$$

Five years later another Austrian physicist, Ludwig Boltzmann (1844–1906), derived this equation, now called the Stefan–Boltzmann equation

$$L = 4\pi R^2 \sigma T_e^4$$

, using the laws of thermodynamics and Maxwell's formula for radiation pressure. Since stars are not perfect blackbodies, we use this equation to define the **effective temperature** T_e of a star's surface.

Because the Sun emits most of its energy at visible wavelengths, and because Earth's atmosphere is transparent at these wavelengths, the evolutionary process of natural selection has produced a human eye sensitive to this wavelength region of the electromagnetic spectrum.(环境造就人类)

3.4.3 The Eve of a New World View

This section draws to a close at the end of the nineteenth century. The physicists and astronomers of the time believed that all of the principles that govern the physical world had finally been discovered. Their scientific world view, the Newtonian paradigm, was the culmination of the heroic, golden age of classical physics that had flourished for over three hundred years. The construction of this paradigm began with the brilliant observations of Galileo and the subtle insights of Newton. Its architecture was framed by Newton's laws, supported by the twin pillars of the conservation of energy and momentum and illuminated by Maxwell's electromagnetic waves. Its legacy was a deterministic description of a universe that ran like clockwork, with wheels turning inside of wheels, all of its gears perfectly meshed. Physics was in danger of becoming a victim of its own success. There were no challenges remaining. All of the great discoveries apparently had been made, and the only task remaining for men and women of science at the end of the nineteenth century was filling in the details

However, as the twentieth century opened, it became increasingly apparent that a crisis was brewing. Physicists were frustrated by their inability to answer some of the simplest questions concerning light. What is the medium through which light waves travel the vast distances between the stars, and what is Earth's speed through this medium? What determines the continuous spectrum of blackbody radiation and the characteristic, discrete colors of tubes filled with hot glowing gases? Astronomers were tantalized by hints of a treasure of knowledge just beyond their grasp.

It took a physicist of the stature of Albert Einstein to topple the Newtonian paradigm and bring about two revolutions in physics. One transformed our ideas about space and time, and the other changed our basic concepts of matter and energy. The rigid clockwork universe of the golden era was found to be an illusion and was replaced by a random universe governed by the laws of probability and statistics. The following four lines aptly summarize the situation. The first two lines were written by the English poet Alexander Pope (1688–1744), a contemporary of Newton; the last two, by Sir J. C. Squire (1884–1958), were penned in 1926.

Nature and Nature's laws lay hid in night:
God said, Let Newton be! and all was light.

It did not last: the Devil howling "Ho!
Let Einstein be!" restored the status quo

3.5 THE QUANTIZATION OF ENERGY

One of the problems haunting physicists at the end of the nineteenth century was their inability to derive from fundamental physical principles the blackbody radiation curve depicted in Fig. 8.

3.5.1 Planck's Function for the Blackbody Radiation Curve

In order to determine the constants a and b while circumventing the ultraviolet catastrophe, Planck employed a clever mathematical trick. He assumed that a standing electromagnetic wave of wavelength λ and frequency $\nu = c/\lambda$ could not acquire just any arbitrary amount of energy. Instead, the wave could have only specific allowed energy values that were integral multiples of a minimum wave energy.

Given this assumption of quantized wave energy with a minimum energy proportional to the frequency of the wave, the entire oven could not contain enough energy to supply even one quantum of energy for the short-wavelength, high-frequency waves. Thus the ultraviolet catastrophe would be avoided. Planck hoped that at the end of his derivation, the constant h could be set to zero; certainly, an artificial constant should not remain in his final result for $B_\lambda(T)$.
Planck's stratagem worked! His formula, now known as the Planck function, agreed wonderfully with experiment, but only if the constant h remained in the equation:

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

3.5.2 The Planck Function and Astrophysics

The Planck function can be used to make the connection between the observed properties of a star (radiant flux, apparent magnitude) and its intrinsic properties (radius, temperature).

3.6 THE COLOR INDEX

The apparent and absolute magnitudes discussed in Section 2, measured over all wavelengths of light emitted by a star, are known as **bolometric magnitudes** and are denoted by $\mathbf{m_{bol}}, \mathbf{M_{bol}}$, respectively. **In practice, however, detectors measure the radiant flux of a star only within a certain wavelength region defined by the sensitivity of the detector**

3.6.1 UBV Wavelength Filters

3.6.2 Color Indices and the Bolometric Correction

Because a color index is the difference between two magnitudes, Eq. (6) shows that it is independent of the star's distance. The difference between a star's bolometric magnitude and its visual magnitude is its **bolometric correction** BC:

$$BC = m_{\text{bol}} - V = M_{\text{bol}} - M_V$$

A sensitivity function $S(\lambda)$ is used to describe the fraction of the star's flux that is detected at wavelength λ. S depends on the reflectivity of the telescope mirrors, the bandwidth of the U, B, and V filters, and the response of the photometer.

Note that although the apparent magnitudes depend on the radius R of the model star and its distance r, the color indices do not, because the factor of $(R/r)^2$ cancels in Eq. (33). Thus the color index is a measure solely of the temperature of a model blackbody star.

3.6.3 The Color–Color Diagram

Figure 11 is a **color–color diagram** showing the relation between the U B and B V color indices for main-sequence stars. **Astronomers face the difficult task of connecting a star's position on a color–color diagram with the physical properties of the star itself**. If stars actually behaved as blackbodies, the color–color diagram would be the straight dashed line shown in Fig. 11. However, stars are not true blackbodies.

Some light is absorbed as it travels through a star's atmosphere, and the amount of light absorbed depends on both the wavelength of the light and the temperature of the star. Other factors also play a role, causing the color indices of main-sequence and supergiant stars of the same temperature to be slightly different.

SUGGESTED READING

The Continuous Spectrum of Light - PROBLEM SET

Chapter 4

The Theory of Special Relativity

4.1 THE FAILURE OF THE GALILEAN TRANSFORMATIONS

4.1.1 The Galilean Transformations

4.1.2 The Michelson–Morley Experiment

4.2 THE LORENTZ TRANSFORMATIONS

The young Albert Einstein (1875–1955; see Fig. 3) enjoyed discussing a puzzle with his friends: What would you see if you looked in a mirror while moving at the speed of light? Would you see your image in the mirror, or not? This was the beginning of Einstein's search for a simple, consistent picture of the universe, a quest that would culminate in his theories of relativity. After much reflection, Einstein finally rejected the notion of an all-pervading ether.

4.2.1 Einstein's Postulates

In 1905 Einstein introduced his two postulates of special relativity3 in a remarkable paper, "On the Electrodynamics of Moving Bodies."

The phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that ... the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable to the former, namely, that light is always propagated in empty space with *a definite speed c which is independent of the state of motion of the emitting body.*

In other words, Einstein's postulates are:

- **The Principle of Relativity** The laws of physics are the same in all inertial reference frames.
- **The Constancy of the Speed of Light** Light moves through a vacuum at a constant speed c that is independent of the motion of the light source.

4.2.2 The Derivation of the Lorentz Transformations

Einstein then went on to derive the equations that lie at the heart of his theory of special relativity, the **Lorentz transformations**.

$$
\begin{aligned}
x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}
\end{aligned}
$$

Lorentz factor: $\gamma \equiv \frac{1}{\sqrt{1-u^2/c^2}}$ may be used to estimate the importance of relativistic effects

4.2.3 Four-Dimensional Spacetime

The Lorentz transformation equations form the core of the theory of special relativity, and they have many surprising and unusual implications. The most obvious surprise is the intertwining roles of spatial and temporal coordinates in the transformations. In the words of Einstein's professor, Hermann Minkowski (1864–1909), "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union between the two will preserve an independent reality." The drama of the physical world unfolds on the stage of a four-dimensional **spacetime**, where events are identified by their spacetime coordinates (x, y, z, t).

4.3 TIME AND SPACE IN SPECIAL RELATIVITY

The implications of this **downfall of universal simultaneity** are far-reaching. The absence of a universal simultaneity means that clocks in relative motion will not stay synchronized. Newton's idea of an absolute universal time that "of itself and from its own nature flows equably without regard to anything external" has been overthrown. Different observers in relative motion will measure different time intervals between the same two events!

4.3.1 Proper Time and Time Dilation

This equation shows the effect of **time dilation** on a moving clock. It says that the time interval between two events is measured differently by different observers in relative motion. The shortest time interval is measured by a clock at rest relative to the two events. This clock measures the **proper time** between the two events. Any other clock moving relative to the two events will measure a longer time interval between them

4.3.2 Proper Length and Length Contraction

4.3.3 Time Dilation and Length Contraction Are Complementary

An effect due to time dilation as measured in one frame may instead be attributed to length contraction as measured in another frame

4.3.4 The Relativistic Doppler Shift

$$v_{\text{obs}} = v_{\text{rest}} \sqrt{\frac{1 - v_r/c}{1 + v_r/c}}$$

where v_r is the radial velocity of the light source.
When astronomers observe a star or galaxy moving away from or toward Earth, the wavelength of the light they receive is shifted toward longer or shorter wavelengths, respectively. If the source of light is moving away from the observer ($v_r > 0$), then $\lambda_{obs} > \lambda_{rest}$. This shift to a longer wavelength is called a **redshift**. Similarly, if the source is moving toward the observer ($v_r < 0$), then there is a shift to a shorter wavelength, a **blueshift**.

Because most of the objects in the universe outside of our own Milky Way Galaxy are moving away from us, redshifts are commonly measured by astronomers. A redshift parameter z is used to describe the change in wavelength; it is defined as:

$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta\lambda}{\lambda_{\text{rest}}}$$

In general, above equation, together with $\lambda = c/\nu$, shows that:

$$z + 1 = \frac{\Delta t_{\text{obs}}}{\Delta t_{\text{rest}}}$$

This expression indicates that if the luminosity of an astrophysical source with redshift parameter $z > 0$ (receding) is observed to vary during a time $\delta t_o bs$, then the change in luminosity occurred over a shorter time $\Delta t_{rest} = \Delta t_{obs}/(z+1)$ in the rest frame of the source
redshift parameter:

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

Example: In its rest frame, the quasar SDSS 1030+0524 produces a hydrogen emission line of wavelength $\lambda_{rest} = 121.6$ nm. On Earth, this emission line is observed to have a wavelength of $\lambda_{obs} = 885.2$ nm. The redshift parameter for this quasar is thus

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = 6.28$$

Using Eq. (36), we may calculate the speed of recession of the quasar:

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

$$\frac{v_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$
$$= 0.963$$

Quasar SDSS 1030+0524 appears to be moving away from us at more than 96% of the speed of light! However, objects that are enormously distant from us, such as quasars, have large apparent recessional speeds due to the overall expansion of the universe. In these cases the increase in the observed wavelength is actually due to the expansion of space itself (which stretches the wavelength of light) rather than being due to the motion of the object through space! This **cosmological redshift** is a consequence of the Big Bang.
This quasar was discovered as a product of the massive Sloan Digital Sky Survey; see Becker, et al. (2001) for further information about this object

4.3.5 The Relativistic Velocity Transformation

The relativistic velocity transform equations do satisfy the second of Einstein's postulates: Light travels through a vacuum at a constant speed that is independent of the motion of the light source.

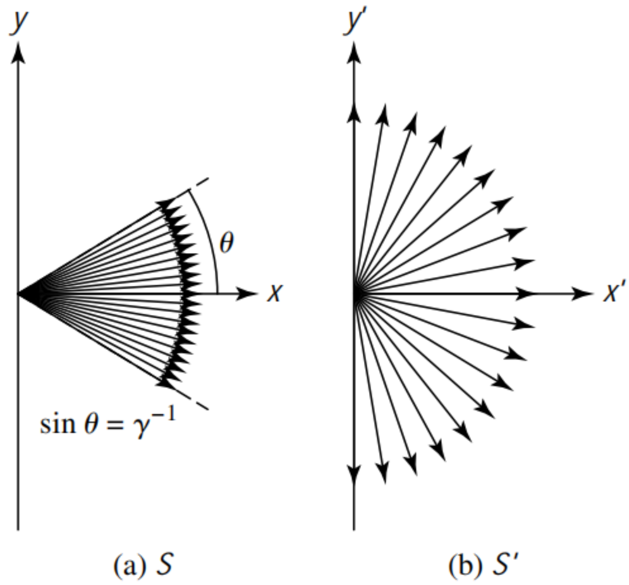

Figure 4.1: Relativistic headlight effect. (a) Frame S, (b) Frame S'.

All of the light emitted into the forward hemisphere, as measured in S' , is concentrated into a narrow cone in the direction of the light source's motion when measured in frame S. Called the **headlight effect**, this result plays an important role in many areas of astrophysics. For example, as relativistic electrons spiral around magnetic field lines, they emit light in the form of **synchrotron radiation**. The radiation is concentrated in the direction of the electron's motion and is strongly plane-polarized. Synchrotron radiation is an important electromagnetic radiation process in the Sun, Jupiter's magnetosphere, pulsars, and active galaxies.

4.4 RELATIVISTIC MOMENTUM AND ENERGY

Up to this point, only relativistic kinematics has been considered. Einstein's theory of special relativity also requires new definitions for the concepts of momentum and energy. The ideas of conservation of linear momentum and energy are two of the cornerstones of physics. According to the Principle of Relativity, if momentum is conserved in one inertial frame of reference, then it must be conserved in all inertial frames.

relativistic momentum vector **p**

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\mathbf{v}$$

In this text, the mass m of a particle is taken to be the same value in all inertial reference frames; it is invariant under a Lorentz transformation, and so there is no reason to qualify the term as a "rest mass." Thus the mass of a moving particle does not increase with increasing speed, although its momentum approaches infinity as $v \to c$.
Also note that the "v" in the denominator is the magnitude of the particle's velocity relative to the observer, not the relative velocity u between two arbitrary frames of reference.

It is left as an exercise to show that $\mathbf{F} = \mathbf{ma}$ is not correct, since at relativistic speeds the force and the acceleration need not be in the same direction!

4.4.1 The Derivation of $\mathbf{E} = \mathbf{mc^2}$

From relativistic momentum and Newton's second law, the expression for the **relativistic kinetic energy**:

$$K = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) = mc^2(\gamma - 1)$$

The right-hand side of this expression for the kinetic energy consists of the difference between two energy terms. The first is identified as the **total relativistic energy E**,

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$$

The second term is an energy that does not depend on the speed of the particle; the particle has this energy even when it is at rest. The term mc^2 is called the **rest energy** of the particle:

$$E_{rest} = mc^2$$

Finally, there is a very useful expression relating a particle's total energy E, the magnitude of its momentum p, and its rest energy mc^2. It states that

$$E^2 = p^2c^2 + m^2c^4$$

This equation is valid even for particles that have no mass, such as photons.

If the momentum of the system of particles is conserved, then the total energy is also conserved, even for inelastic collisions in which the kinetic energy of the system is reduced. The kinetic energy lost in the inelastic collisions goes into increasing the rest energy, and hence the mass, of the particles. This increase in rest energy allows the total energy of the system to be conserved. Mass and energy are two sides of the same coin; one can be transformed into the other.

4.4.2 The Derivation of Relativistic Momentum

SUGGESTED READING

The Theory of Special Relativity - PROBLEM SET

The temporal order of cause and effect is preserved by the Lorentz transformation equations.
The spacetime interval Δs is said to be invariant under a Lorentz transformation

Chapter 5

The Interaction of Light and Matter

5.1 SPECTRAL LINES

In 1835 a French philosopher, Auguste Comte (1798–1857), considered the limits of human knowledge. In his book Positive Philosophy, Comte wrote of the stars, "We see how we may determine their forms, their distances, their bulk, their motions, but we can never know anything of their chemical or mineralogical structure." Thirty-three years earlier, however, William Wollaston (1766–1828), like Newton before him, passed sunlight through a prism to produce a rainbow-like spectrum. He discovered that a number of dark **spectral lines** were superimposed on the continuous spectrum where the Sun's light had been absorbed at certain discrete wavelengths. By 1814, the German optician Joseph von Fraunhofer (1787–1826) had cataloged 475 of these dark lines (today called **Fraunhofer lines**) in the solar spectrum. While measuring the wavelengths of these lines, Fraunhofer made the first observation capable of proving Comte wrong. Fraunhofer determined that the wavelength of one prominent dark line in the Sun's spectrum corresponds to the wavelength of the yellow light emitted when salt is sprinkled in a flame. The new science of spectroscopy was born with the identification of this sodium line

5.1.1 Kirchhoff's Laws

In 1860 Kirchhoff and Bunsen published their classic work Chemical Analysis by Spectral Observations, in which they developed the idea that every element produces its own pattern of spectral lines and thus may be identified by its unique spectral line "fingerprint." Kirchhoff summarized the production of spectral lines in three laws, which are now known as **Kirchhoff's laws**.

- A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines.
  (In the first of Kirchhoff's laws, "hot" actually means any temperature above 0 K. However, according to Wien's displacement law a temperature of several thousand degrees K is required for λ_{max} to fall in the visible portion of the electromagnetic spectrum. it is the opacity or optical depth of the gas that is responsible for the continuous blackbody spectrum.)

- A hot, diffuse gas produces bright spectral lines (**emission lines**).

- A cool, diffuse gas in front of a source of a continuous spectrum produces dark spectral lines (**absorption lines**) in the continuous spectrum

5.1.2 Applications of Stellar Spectra Data

An immediate application of these results was the identification of elements found in the Sun and other stars. A new element previously unknown on Earth, helium, was discovered spectroscopically on the Sun in 1868; it was not found on Earth until 1895.

Another rich line of investigation was pursued by measuring the Doppler shifts of spectral lines, which can be utilized to determine radial velocities. By 1887 the radial velocities of Sirius, Procyon, Rigel, and Arcturus had been measured with an accuracy of a few kilometers per second.

The average speed of stars in the solar neighborhood is about 25 km s^{-1} . In reality, the measurement of a star's radial velocity is complicated by the 29.8 km s^{-1} motion of Earth around the Sun, which causes the observed wavelength λ_{obs} of a spectral line to vary sinusoidally over the course of a year. This effect of Earth's speed may be easily compensated for by subtracting the component of Earth's orbital velocity along the line of sight from the star's measured radial velocity

5.1.3 Spectrographs

Modern methods can measure radial velocities with an accuracy of better than $\pm 3ms^{-1}$! Today astronomers use spectrographs to measure the spectra of stars and galaxies; After passing through a narrow slit, the starlight is collimated by a mirror and directed onto a diffraction grating. A diffraction grating is a piece of glass onto which narrow, closely spaced lines have been evenly ruled (typically several thousand lines per millimeter); the grating may be made to transmit the light (a transmission grating) or reflect the light (a reflection grating). In either case, the grating acts like a long series of neighboring double slits. Different wavelengths of light have their maxima occurring at different angles θ given by the following equation:

$$d \sin \theta = n\lambda (n = 0, 1, 2, ...)$$

where d is the distance between adjacent lines of the grating, n is the order of the spectrum, and θ is measured from the line normal (or perpendicular) to the grating. (n = 0 corresponds to $\theta = 0$ for all wavelengths, so the light is not dispersed into a spectrum in this case.) The spectrum is then focused onto a photographic plate or electronic detector for recording.
(Measuring the radial velocities of stars in binary star systems allows the masses of the stars to be determined. The same methods have now been used to detect numerous extrasolar planets.)
Modern depictions of spectra are typically shown as **plots of flux as a function of wavelength**.

The ability of a spectrograph to resolve two closely spaced wavelengths separated by an amount $\Delta\lambda$ depends on the order of the spectrum, n, and the total number of lines of the grating that are illuminated, N. The smallest difference in wavelength that the grating can resolve is

$$\Delta\lambda = \frac{\lambda}{nN}$$

where λ is either of the closely spaced wavelengths being measured. The ratio $\lambda/\Delta\lambda$ is the **resolving power** of the grating. (In some cases, the resolving power of a spectrograph may be determined by other factors—for example, the slit width.)

Astronomers recognized the great potential for uncovering the secrets of the stars in the empirical rules that had been obtained for the spectrum of light: Wien's law, the Stefan– Boltzmann equation, Kirchhoff's laws, and the new science of spectroscopy. By 1880 Gustav Wiedemann (1826–1899) found that a detailed investigation of the Fraunhofer lines could reveal the temperature, pressure, and density of the layer of the Sun's atmosphere that produces the lines. The splitting of spectral lines by a magnetic field was discovered by Pieter Zeeman (1865–1943) of the Netherlands in 1897, raising the possibility of measuring stellar magnetic fields. But a serious problem blocked further progress: However impressive, these results lacked the solid theoretical foundation required for the interpretation of stellar spectra. For example, the absorption lines produced by hydrogen are much stronger for Vega than for the Sun. Does this mean that Vega's composition contains significantly more hydrogen than the Sun's? The answer is no, but how can this information be gleaned from the dark absorption lines of a stellar spectrum recorded on a photographic plate? The answer required a new understanding of

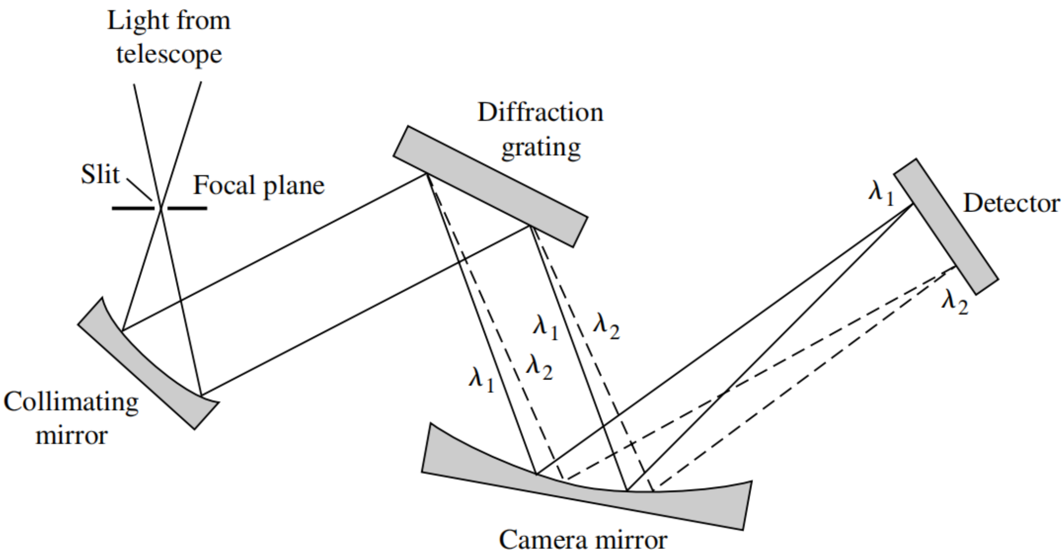

Figure 5.1: Spectrograph.

the nature of light itself.

5.2 PHOTONS

Despite Heinrich Hertz's absolute certainty in the wave nature of light, the solution to the riddle of the continuous spectrum of blackbody radiation led to a complementary description, and ultimately to new conceptions of matter and energy. Planck's constant h is the basis of the modern description of matter and energy known as **quantum mechanics**. Today h is recognized as a fundamental constant of nature, like the speed of light c and the universal gravitational constant G. Although Planck himself was uncomfortable with the implications of his discovery of energy quantization, quantum theory was to develop into what is today a spectacularly successful description of the physical world. The next step forward was taken by Einstein, who convincingly demonstrated the reality of Planck's quantum bundles of energy.

5.2.1 The Photoelectric Effect

When light shines on a metal surface, electrons are ejected from the surface, a result called the **photoelectric effect**.

A surprising feature of the photoelectric effect is that the value of K_{max} does not depend on the brightness of the light shining on the metal. Increasing the inten-

sity of a monochromatic light source will eject more electrons but will not increase their maximum kinetic energy. Instead, K_{max} varies with the frequency of the light illuminating the metal surface.
This puzzling frequency dependence is nowhere to be found in Maxwell's classic description of electromagnetic waves. The equation for the Poynting vector admits no role for the frequency in describing the energy carried by a light wave

Einstein's bold solution was to take seriously Planck's assumption of the quantized energy of electromagnetic waves.
According to Einstein's explanation of the photoelectric effect, the light striking the metal surface consists of a stream of massless particles called **photons**, The energy of a single photon of frequency ν and wavelength λ is just the energy of Planck's quantum of energy:

$$E_{\text{photon}} = hv = \frac{hc}{\lambda}$$

This means that the 100W light bulb emits 2.52×10^{20} photons per second .As this huge number illustrates, with so many photons nature does not appear "grainy." We see the world as a continuum of light, illuminated by a flood of photons.

The photoelectric effect established the reality of Planck's quanta. Albert Einstein was awarded the 1921 Nobel Prize, not for his theories of special and general relativity, but "for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect."
Partly in recognition of his determination of an accurate value of Planck's constant h the American physicist Robert A. Millikan (1868–1953) also received a Nobel Prize (1923) for his work on the photoelectric effect.
Today astronomers take advantage of the quantum nature of light in various instruments and detectors, such as CCDs (charge-coupled devices).

5.2.2 The Compton Effect

In 1922, the American physicist Arthur Holly Compton (1892–1962) provided the most convincing evidence that light does in fact manifest its particle-like nature when interacting with matter.
Compton measured the change in the wavelength of X-ray photons as they were scattered by free electrons.
Today, this change in wavelength is known as the **Compton effect**

5.3 THE BOHR MODEL OF THE ATOM

The pioneering work of Planck, Einstein, and others at the beginning of the twentieth century revealed the **wave–particle duality of light**. Light exhibits its wave properties as it propagates through space, as demonstrated by its double-slit interference pattern. On the other hand, light manifests its particle nature when it interacts with matter, as in the photoelectric and Compton effects. Planck's formula describing the energy distribution of blackbody radiation explained many of the features of the continuous spectrum of light emitted by stars. But what physical process was responsible for the dark absorption lines scattered throughout the continuous spectrum of a star, or for the bright emission lines produced by a hot, diffuse gas in the laboratory?

5.3.1 The Structure of the Atom

Rutherford directed high-speed alpha particles (now known to be helium nuclei) onto thin metal foils. He was amazed to observe that a few of the alpha particles were bounced backward by the foils instead of plowing through them with only a slight deviation. Rutherford later wrote: "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." Such an event could occur only as the result of a single collision of the alpha particle with a minute, massive, positively charged nucleus. Rutherford calculated that the radius of the nucleus was 10,000 times smaller than the radius of the atom itself, **showing that ordinary matter is mostly empty space**!

5.3.2 The Wavelengths of Hydrogen

The experimental data were abundant. The wavelengths of 14 spectral lines of hydrogen had been precisely determined.
In 1885 a Swiss school teacher, Johann Balmer (1825–1898), had found, by trial and error, a formula to reproduce the wavelengths of these spectral lines of hydrogen, today called the Balmer series or Balmer lines. Balmer's formula was very accurate, to within a fraction of a percent.
Furthermore, Balmer realized that since $2^2 = 4$, his formula could be generalized, many nonvisible spectral lines of hydrogen were found later, just as Balmer had predicted!

Yet all of this was sheer numerology, with no foundation in the physics of the day. Physicists were frustrated by their inability to construct a model of even this simplest of atoms.

A planetary model of the hydrogen atom, consisting of a central proton and one electron held together by their mutual electrical attraction, should have been most amenable to analysis. However, a model consisting of a single electron and proton moving around their common center of mass suffers from a basic instability. **According to Maxwell's equations of electricity and magnetism, an accelerating electric charge emits electromagnetic radiation. The orbiting electron should thus lose energy by emitting light with a continuously increasing frequency (the orbital frequency) as it spirals down into the nucleus**. This theoretical prediction of a continuous spectrum disagreed with the discrete emission lines actually observed. Even worse was the calculated timescale: The electron should plunge into the nucleus in only 10^{-8} s. Obviously, matter is stable over much longer periods of time!
电子,原子核

5.3.3 Bohr's Semiclassical Atom

Theoretical physicists hoped that the answer might be found among the new ideas of photons and quantized energy

A Danish physicist, Niels Bohr (1885–1962; see Fig. 4) came to the rescue in 1913 with a daring proposal. The dimensions of Planck's constant, $J \times s$, are equivalent to $kg \times ms^{-1} \times m$, the units of angular momentum. Perhaps the angular momentum of the orbiting electron was quantized. This quantization had been previously introduced into atomic models by the British astronomer J. W. Nicholson. Although Bohr knew that Nicholson's models were flawed, he recognized the possible significance of the quantization of angular momentum. Just as an electromagnetic wave of frequency ν could have the energy of only an integral number of quanta, $E = nh\nu$, suppose that the value of the angular momentum of the hydrogen atom could assume only integral multiples of Planck's constant divided by 2π: $L = nh/2\pi = n\hbar$. Bohr hypothesized that in orbits with precisely these allowed values of the angular momentum, the electron would be stable and would not radiate in spite of its centripetal acceleration. What would be the result of such a bold departure from classical physics?

$E = U/2 = -K$. Because the kinetic energy must be positive, the total energy E is negative. This merely indicates that the electron and the proton are bound. To **ionize** the atom (that is, to remove the proton and electron to an infinite separation), an amount of energy of magnitude $|E|$ (or more) must be added to the atom.

Solving this equation for the radius r shows that the only values allowed by Bohr's quantization condition are

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}n^2 = a_0 n^2$$

According to Bohr's hypothesis, when the electron is in one of these orbits, the atom is stable and emits no radiation.
The allowed energies of the Bohr atom are:

$$E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -13.6\text{eV}\frac{1}{n^2}$$

The integer n, known as the **principal quantum number**, completely determines the characteristics of each orbit of the Bohr atom.

If the electron does not radiate in any of its allowed orbits, then what is the origin of the spectral lines observed for hydrogen? Bohr proposed that a photon is emitted or absorbed when an electron makes a transition from one orbit to another.

After the quantum revolution, the physical processes responsible for Kirchhoff's laws (discussed in Section 1) finally became clear:

- A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines. This is the continuous spectrum of blackbody radiation emitted at any temperature above absolute zero and described by the Planck functions $B_\lambda(T)$ and $B_\nu(T)$. The wavelength λ_{max} at which the Planck function $B_\lambda(T)$ obtains its maximum value is given by Wien's displacement law

- A hot, diffuse gas produces bright emission lines. Emission lines are produced when an electron makes a downward transition from a higher orbit to a lower orbit. The energy lost by the electron is carried away by a single photon. For example, the hydrogen Balmer emission lines are produced by electrons "falling" from higher orbits down to the $n = 2$ orbit.

- A cool, diffuse gas in front of a source of a continuous spectrum produces dark absorption lines in the continuous spectrum. Absorption lines are produced when an electron makes a transition from a lower orbit to a higher orbit. If an incident photon in the continuous spectrum has exactly the right amount of energy, equal to the difference in energy between a higher orbit and the electron's initial orbit, the photon is absorbed by the atom and the electron makes an upward transition to that higher orbit. For example, the hydrogen Balmer absorption lines are produced by atoms absorbing photons that cause electrons to make transitions from the $n = 2$ orbit to higher orbits;

In fact, the electron orbits are not circular. They are not even orbits at all, in the classical sense of an electron at a precise location moving with a precise velocity. Instead, on an atomic level, nature is "fuzzy", with an attendant uncertainty that cannot be avoided. It was fortunate that Bohr's model, with all of its faults, led to the correct values for the energies of the orbits and to a correct interpretation of the formation of spectral lines. This intuitive, easily imagined model of the atom is what most physicists and astronomers have in mind when they visualize atomic processes.

5.4 QUANTUM MECHANICS AND WAVE–PARTICLE DUALITY

The last act of the quantum revolution began with the musings of a French prince, Louis de Broglie (1892–1987). Wondering about the recently discovered wave–particle duality for light, he posed a profound question: If light (classically thought to be a wave) could exhibit the characteristics of particles, might not particles sometimes manifest the properties of waves

5.4.1 de Broglie's Wavelength and Frequency

In his 1927 Ph.D. thesis, de Broglie extended the wave–particle duality to all of nature.
The **de Broglie wavelength** and **frequency** describe not only massless photons but massive electrons, protons, neutrons, atoms, molecules, people, planets, stars, and galaxies as well. This seemingly outrageous proposal of matter waves has been confirmed in countless experiments. Figure 9 shows the interference pattern produced by electrons in a double-slit experiment. Just as Thomas Young's double-slit experiment established the wave properties of light, the electron double-slit experiment can be explained only by the wave-like behavior of electrons, with each electron propagating through both slits. The wave–particle duality applies to everything in the physical world; **everything exhibits its wave properties in its propagation and manifests its particle nature in its interactions**.

Just what are the waves that are involved in the wave-particle duality of nature? In a double-slit experiment, each photon or electron must pass through both slits, since the interference pattern is produced by the constructive and destructive interference of the two waves. Thus the wave cannot convey information about where the photon or electron is, but only about where it may be. The wave is one of *probability*, and its amplitude is denoted by the Greek letter Ψ (psi). The square of the wave amplitude, $|\Psi|^2$, at a certain location describes the probability of finding the photon or electron at that location. In the double-slit experiment, photons or electrons are never found where the waves from slits 1 and 2 have destructively interfered—that is, where $|\Psi_1 + \Psi_2|^2 = 0$

5.4.2 Heisenberg's Uncertainty Principle

The wave attributes of matter lead to some unexpected conclusions of paramount importance for the science of astronomy.

This is nature's intrinsic trade-off: The uncertainty in a particle's position, Δx, and the uncertainty in its momentum, Δp, are inversely related. As one decreases, the other must increase. This fundamental inability of a particle to simultaneously have a well-defined position and a well-defined momentum is a direct result of the wave–particle duality of nature. A German physicist, Werner Heisenberg (1901–1976), placed this inherent "fuzziness" of the physical world in a firm theoretical framework. He demonstrated that the uncertainty in a particle's position multiplied by the uncertainty in its momentum must be at least as large as $\hbar/2$:

$$\Delta x \Delta p \geq \frac{1}{2}\hbar$$

Today this is known as **Heisenberg's uncertainty principle**. The equality is rarely realized in nature A similar statement relates the uncertainty of an energy

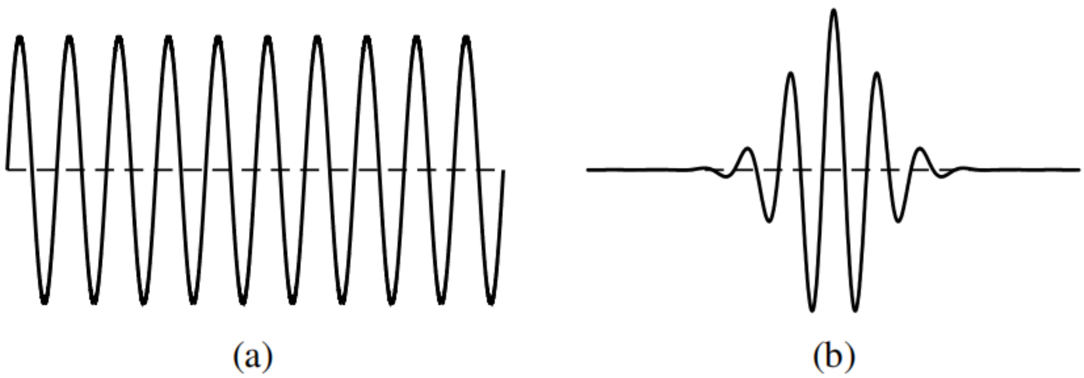

Figure 5.2: Two examples of a probability wave, Ψ: (a) a single sine wave (能量确定但位置不确定) and (b) a pulse composed of many sine waves(位置限制在更小的范围, 但能量不确定(只是许多波长中的一个, 至于是哪个波长确定不了)) 海森堡不确定性原理可以从傅立叶级数展开的角度去分析!

measurement, ΔE, and the time interval, Δt, over which the energy measurement is taken:

$$\Delta E \Delta t \approx \hbar$$

As the time available for an energy measurement increases, the inherent uncertainty in the result decreases.

Imagine an electron confined within a region of space the size of a hydrogen atom. We can estimate the minimum speed and kinetic energy of the electron using Heisenberg's uncertainty principle.

This is in good agreement with the kinetic energy of the electron in the ground state of the hydrogen atom. An electron confined to such a small region must move rapidly with at least this speed and this energy. This subtle quantum effect is responsible for supporting white dwarf and neutron stars against the tremendous inward pull of gravity.

5.4.3 Quantum Mechanical Tunneling

In general, when a classical wave such as a water or light wave enters a medium through which it cannot propagate, it becomes evanescent and its amplitude decays exponentially with distance. This total internal reflection can in fact be frustrated

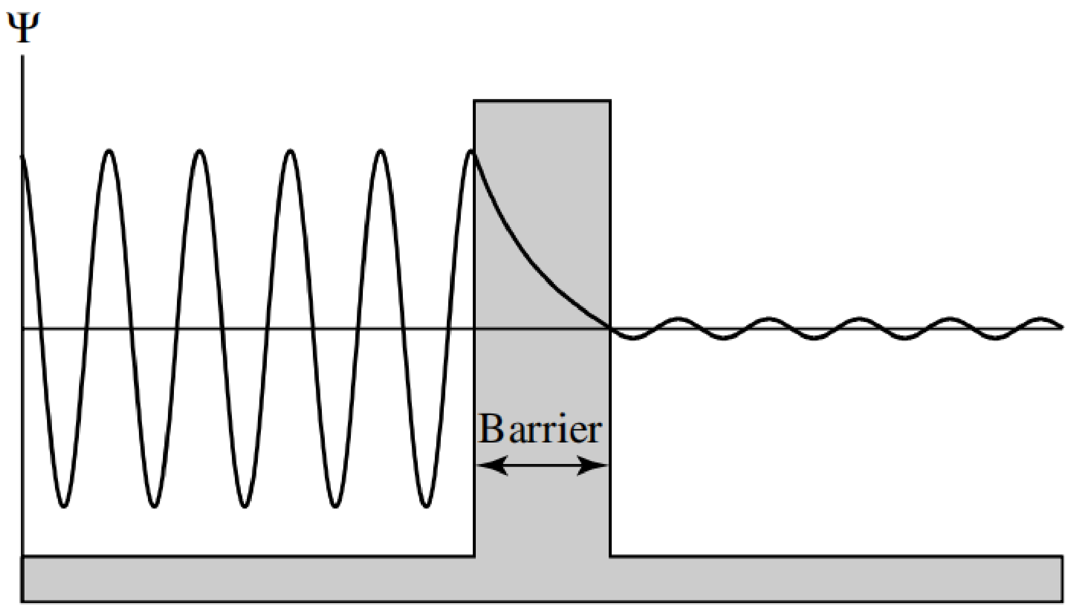

Figure 5.3: Quantum mechanical tunneling (barrier penetration) of a particle traveling to the right

by placing another prism next to the first prism so that their surfaces nearly (but not quite) touch. Then the evanescent wave in the air may enter the second prism before its amplitude has completely died away. The electromagnetic wave once again becomes oscillatory upon entering the glass, and so the ray of light has traveled from one prism to another without passing through the air gap between the prisms. In the language of particles, photons have **tunneled** from one prism to another without traveling in the space between them.

The wave–particle duality of nature implies that particles can also tunnel through a region of space (a barrier) in which they cannot exist classically, as illustrated in Fig. 11. The barrier must not be too wide (not more than a few particle wavelengths) if tunneling is to take place; otherwise, the amplitude of the evanescent wave will have declined to nearly zero. This is consistent with Heisenberg's uncertainty principle, which implies that a particle's location cannot be determined with an uncertainty that is less than its wavelength. Thus, if the barrier is only a few wavelengths wide, the particle may suddenly appear on the other side of the barrier. **Barrier penetration** is extremely important in radioactive decay, where alpha particles tunnel out of an atom's nucleus; in modern electronics, where it is the basis for the "tunnel diode"; and inside stars, where the rates of nuclear fusion reactions depend upon tunneling.

5.4.4 Schrödinger's Equation and the Quantum Mechanical Atom

What are the implications for Bohr's model of the hydrogen atom? Heisenberg's uncertainty principle does not allow classical orbits, with their simultaneously precise values of the electron's position and momentum. Instead, **the electron orbitals must be imagined as fuzzy clouds of probability**, with the clouds being more "dense" in regions where the electron is more likely to be found. In 1925 a complete break from classical physics was imminent, one that would fully incorporate de Broglie's matter waves. Maxwell's The Interaction of Light and Matter equations of electricity and magnetism can be manipulated to produce a wave equation for the electromagnetic waves that describe the propagation of photons. Similarly, a wave equation discovered in 1926 by Erwin Schrödinger (1877–1961), an Austrian physicist, led to a true **quantum mechanics**, the quantum analog of the classical mechanics that originated with Galileo and Newton. The Schrödinger equation can be solved for the probability waves that describe the allowed values of a particle's energy, momentum, and so on, as well as the particle's propagation through space. In particular, the Schrödinger equation can be solved analytically for the hydrogen atom, giving exactly the same set of allowed energies as those obtained by Bohr. However, in addition to the principal quantum number n, Schrödinger found that two additional quantum numbers, ell and m_ℓ, are required for a complete description of the electron orbitals. These additional numbers describe the angular momentum vector, **L**, of the atom. Instead of the quantization used by Bohr, $L = n\hbar$, the solution to the Schrödinger equation shows that the permitted values of the magnitude of the angular momentum L are actually

$$L = \sqrt{\ell(\ell+1)}\hbar$$

where $\ell = 0, 1, 2, ..., n-1$, and **n is the principal quantum number that determines the energy**.

Different orbitals, labeled by different values of ℓ and m_ℓ, are said to be **degenerate** if they have the same value of the principal quantum number n and so have the same energy. Electrons making a transition from a given orbital to one of several degenerate orbitals will produce the same spectral line, because they experience the same change in energy However, **the atom's surroundings may**

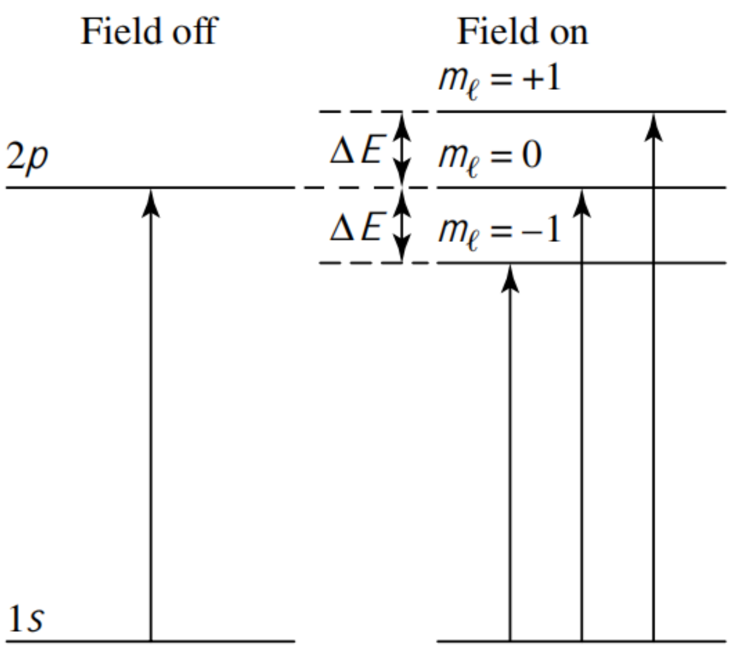

Figure 5.4: Splitting of absorption lines by the Zeeman effect.

single out one spatial direction as being different from another. For example, an electron in an atom will feel the effect of an external magnetic field. The magnitude of this effect will depend on the $2\ell + 1$ possible orientations of the electron's motion, as given by m_ℓ, and the magnetic field strength, B, where the units of B are teslas (T). As the electron moves through the magnetic field, the normally degenerate orbitals acquire slightly different energies. Electrons making a transition between these formerly degenerate orbitals will thus produce spectral lines with slightly different frequencies. The splitting of spectral lines in a weak magnetic field is called the **Zeeman effect**. The three frequencies of the split lines in the simplest case (called the normal Zeeman effect) are

$$v = v_0 \quad \text{and} \quad v_0 \pm \frac{eB}{4\pi\mu}$$

where ν_0 is the frequency of the spectral line in the absence of a magnetic field and μ is the reduced mass. Although the energy levels are split into $2\ell+1$ components, electron transitions involving these levels produce just three spectral lines with different polarizations. Viewed from different directions, it may happen that not all three lines will be visible. For example, when looking parallel to the magnetic field (as when looking down on a sunspot), the unshifted line of frequency ν_0 is absent.

Thus the Zeeman effect gives astronomers a probe of the magnetic fields observed around sunspots and on other stars. Even if the splitting of the spectral line is too small to be directly detected, the different polarizations across the closely spaced components can still be measured and the magnetic field strength deduced

Example: Interstellar clouds may contain very weak magnetic fields, as small as $B \approx 2 \times 10^{-10}$ T. Nevertheless, astronomers have been able to measure this magnetic field. Using radio telescopes, they detect the variation in polarization that occurs across the blended Zeeman components of the absorption lines that are produced by these interstellar clouds of hydrogen gas. The change in frequency, $\Delta\nu$, produced by a magnetic field of this magnitude can be calculated by using the mass of the electron, m_e, for the reduced mass μ:

$$\Delta v = \frac{eB}{4\pi m_e} = 2.8\text{Hz}$$

a minute change. The total difference in frequency from one side of this blended line to the other is twice this amount, or 6 Hz. For comparison, the frequency of the radio wave emitted by hydrogen with $\lambda = 21$ cm is $\nu = c/\lambda = 1.4 \times 10^9$ Hz, 250 million times larger!

5.4.5 Spin and the Pauli Exclusion Principle

Attempts to understand more complicated patterns of magnetic field splitting (the anomalous Zeeman effect), usually involving an even number of unequally spaced spectral lines, led physicists in 1925 to discover a fourth quantum number. In addition to its orbital motion, the electron possesses a **spin**. This is not a classical top-like rotation but purely a quantum effect that endows the electron with a spin angular momentum S. S is a vector of constant magnitude

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar$$

with a z-component $S_z = m_S\hbar$. The only values of the fourth quantum number, m_S, are $\pm\frac{1}{2}$

With each orbital, or quantum state, labeled by four quantum numbers, **physicists wondered how many electrons in a multielectron atom could occupy the same quantum state**. The answer was supplied in 1925 by an Austrian theoretical physicist, Wolfgang Pauli (1900–1958): No two electrons can occupy the same quantum state. The Pauli exclusion principle, that **no two electrons can share the same set of four quantum numbers**, explained the electronic structure of atoms, thereby providing an explanation of the properties of the periodic table of the elements, the well-known chart from any introductory chemistry text. Despite this success, Pauli was unhappy about the lack of a firm theoretical understanding of electron spin. Spin was stitched onto quantum theory in an ad hoc manner, and the seams showed. Pauli lamented this patchwork theory and asked, “How can one avoid despondency if one thinks of the anomalous Zeeman effect?

The final synthesis arrived in 1928 from an unexpected source. A brilliant English theoretical physicist, Paul Adrien Maurice Dirac (1902–1984), was working at Cambridge to combine Schrödinger’s wave equation with Einstein’s theory of special relativity. **When he finally succeeded in writing a relativistic wave equation for the electron, he was delighted to see that the mathematical solution automatically included the spin of the electron.** It also explained and extended the Pauli exclusion principle by dividing the world of particles into two fundamental groups: **fermions and bosons**. **Fermions** are particles such as electrons, protons, and neutrons that have a spin of $\frac{1}{2}\hbar$ (or an odd integer times $\frac{1}{2}\hbar$, such as $\frac{3}{2}\hbar$, $\frac{5}{2}\hbar$,...). Fermions obey the Pauli exclusion principle, so no two fermions of the same type can have the same set of quantum numbers. The exclusion principle for fermions, along with Heisenberg’s uncertainty relation, explains the structure of white dwarfs and neutron stars. Bosons are particles such as photons that have an integral spin of 0, $\hbar$, $2\hbar$, $3\hbar$,... **Bosons** do not obey the Pauli exclusion principle, so any number of bosons can occupy the same quantum state.

As a final bonus, the Dirac equation predicted the existence of antiparticles. A particle and its antiparticle are identical except for their opposite electric charges

and magnetic moments. Pairs of particles and antiparticles may be created from the energy of gamma-ray photons (according to $E = mc^2$). Conversely, particle–antiparticle pairs may annihilate each other, with their mass converted back into the energy of two gamma-ray photons.
Pair creation and annihilation play a major role in the evaporation of black holes.

5.4.6 The Complex Spectra of Atoms

With the full list of four quantum numbers (n, ℓ, m_ℓ, and m_S) that describe the detailed state an each electron in an atom, the number of possible energy levels increases rapidly with the number of electrons. When we take into account the additional complications of external magnetic fields, and the electromagnetic interactions between the electrons themselves and between the electrons and the nucleus, the spectra can become very complicated indeed.

Although energy levels exist for electrons with various combinations of quantum numbers, it is not always easy for an electron to make a transition from one quantum state with a specific set of quantum numbers to another quantum state. In particular, Nature imposes a set of **selection rules** that restrict certain transitions. **allowed/forbidden trainsitions** **现在还只是发现了这种现象，有没有物理理论上的解释？**

Although forbidden transitions may occur, they require much longer times if they are to occur with any significant probability. Since collisions between atoms trigger transitions and can compete with spontaneous transitions, very low gas densities are required for measurable intensities to be observed from forbidden transitions. Such environments do exist in astronomy, such as in the diffuse interstellar medium or in the outer atmospheres of stars.

The revolution in physics started by Max Planck culminated in the quantum atom and gave astronomers their most powerful tool: a theory that would enable them to analyze the spectral lines observed for stars, galaxies, and nebulae. Different atoms, and combinations of atoms in molecules, have orbitals of distinctly different energies; thus they can be identified by their spectral line "fingerprints." The specific spectral lines produced by an atom or molecule depend on which orbitals are occupied by electrons. This, in turn, depends on its surroundings: the temperature, density, and pressure of its environment. These and other factors, such as the strength of a surrounding magnetic field, may be determined by a careful examination of spectral lines.

SUGGESTED READING

The Interaction of Light and Matter - PROBLEM SET

- 4. The photoelectric effect can be an important heating mechanism for the grains of dust found in interstellar clouds (see Section 12.1). The ejection of an electron leaves the grain with a positive charge, which affects the rates at which other electrons and ions collide with and stick to the grain to produce the heating. This process is particularly effective for ultraviolet photons ($\lambda \approx 100$ nm) striking the smaller dust grains. If the average energy of the ejected electron is about 5 eV, estimate the work function of a typical dust grain.

$$\phi = \frac{hc}{\lambda} - K_{\max} = \frac{1240\text{eVnm}}{100\text{nm}} - 5\text{eV} = 7.4\text{eV}$$

- 7. Verify that the units of Planck's constant are the units of angular momentum
Planck's constant has units of

$$\text{Js} = \left[\text{kgm}^2\text{s}^{-2}\right]\text{s} = \text{kg}\left[\text{ms}^{-1}\right]\text{m}$$

which are the units of angular momentum (mvr)

- 14. A white dwarf is a very dense star, with its ions and electrons packed extremely close together. Each electron may be considered to be located within a region of size $\Delta x \approx 1.5 \times 10^{-12}$ m. Use Heisenberg's uncertainty principle, to estimate the minimum speed of the electron. Do you think that the effects of relativity will be important for these stars?
As shown in Example 5.4.2, the minimum speed of a particle (in this case, an electron) can be estimated using Heisenberg's uncertainty relation, Eq. (5.19), as:

$$v_{\min} = \frac{p_{\min}}{m_e} \approx \frac{\Delta p}{m_e} \approx \frac{\hbar}{m_e \Delta x}$$

With $\Delta x \approx 1.5 \times 10^{-12}$ m, the minimum speed of the electron is $v_{min} \approx 7.7 \times 10^7 ms^{-1}$, about 26% of the speed of light. Relativistic effects are important for white dwarf stars.

- 15. natural broadening

- degeneracy of energy level n is $2n^2$

COMPUTER PROBLEM

- 18. One of the most important ideas of the physics of waves is that any complex waveform can be expressed as the sum of the harmonics of simple cosine and sine waves

Chapter 6

Telescopes

6.1 BASIC OPTICS

From the beginning, astronomy has been an observational science.

6.1.1 Refraction and Reflection

The focal length of a given thin lens can be calculated directly from its index of refraction and geometry. If we assume that both surfaces of the lens are spheroidal then it can be shown that the focal length f; is given by the **lensmaker's formula**,

$$\frac{1}{f_\lambda} = (n_\lambda - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where n_λ is the index of refraction of the lens and R_1 and R_2, are the radii of curvature of each surface taken to be positive if the specific surface is convex and negative if it is concave.

Converging mirrors are generally used as the main mirrors in reflecting telescopes, although either diverging or flat mirrors may be used in other parts of the optical system.

6.1.2 The Focal Plane

The focal plane is defined as the plane passing through the focal point and oriented perpendicular to the optical axis of the system.

The differential relation known as the **plate scale**:

$$\frac{d\theta}{dy} = \frac{1}{f}$$

which connects the angular separation of the objects with the linear separation of their images at the focal plane. As the focal length of the lens is increased, the linear separation of the image of two point sources separated by an angle θ also increase.

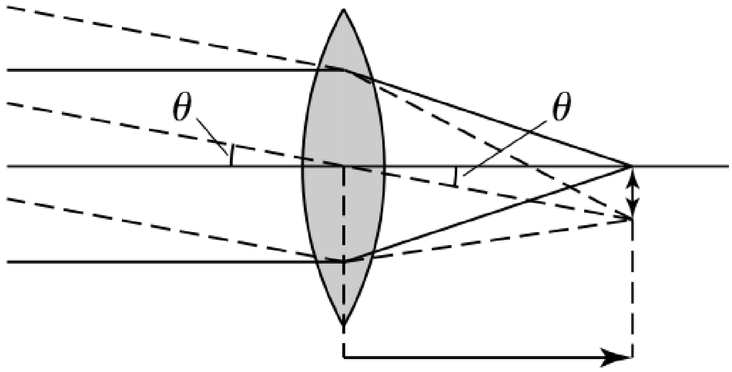

Figure 6.1: The plate scale, determined by the focal length of the optical system

6.1.3 Resolution and the Rayleigh Criterion

Unfortunately, the ability to "see" two objects in space that have a small angular separation θ is not simply a matter of choosing a focal length sufficiently long to produce the necessary plate scale. A fundamental limit exists in our ability to resolve those objects. This limitation is due to diffraction produced by the advancing wavefronts of light coming from those objects. This phenomenon is closely related to the well-known single-slit diffraction pattern, which is similar to the Young double-slit interference pattern.
Airy disk, Rayleigh criterion.

6.1.4 Seeing

The turbulent nature of Earth's atmosphere. Local changes in atmospheric temperature and density over distances ranging from centimeters to meters create regions where the light is refracted in nearly random directions, causing the image of a point source to become blurred.
Since virtually all stars effectively appear as point sources, even when viewed through the largest telescopes) atmospheric turbulence tinkling" of stellar images. The quality of the image of a stellar point source at a given observing location at a specific time is referred to as **seeing**.

6.1.5 Aberrations

Both lens and mirror systems suffer from inherent image distortions known as aberrations. Often these aberrations are common to both types of systems, but **chromatic aberration** is unique to refracting telescopes. The problem stems from the fact that the focal length of a lens is wavelength-dependent.
The problem of chromatic aberration can be diminished somewhat by the addition of correcting lenses.

Several aberrations result from the shape of the reflecting or refracting surface(s). Although it is easier, and therefore cheaper, to grind lenses and mirrors into spheroids, not all areas of these surfaces will focus a parallel set of light rays to a single point. This effect, known as **spherical aberration**, can be overcome by producing carefully designed optical surfaces (paraboloids).

The cause of HST's initial imaging problems is a classical case of spherical aberration. Some of HST's original spherical aberration was compensated for by th e use of computer programs designed to analyze the images produced by the flawed optical system and mathematically create corrected versions.
Today the spherical aberration problems of HST is only a bad memory of what can go wrong.

Even when paraboloids are used, mirrors are not necessarily free from aberrations. **Coma** produces elongated images of point sources that lie off the optical axis, because the focal lengths of paraboloids are a function of 6, the angle between the direction of an incoming light ray and the optical axis. **Astigmatism** is a defect that derives from having different parts of a lens or mirror converge an image at slightly different locations on the focal plane. When a lens or mirror is designed to correct for astigmatism, **curvature of field** can then be a problem. Curvature of field is due to the focusing of images on a curve rather than on a plane. Yet another potential difficulty occurs when the plate scale (Eq. 4) depends on the distance from the optical axis; this effect is referred to as **distortion of field**.

6.1.6 The Brightness of an Image

The image intensity is identical to the object intensity, independent of the area of the aperture. This result is completely analogous to the simple observation that a wall does not appear to get brighter when the observer walks toward it.

The concept that describes the effect of the light-gathering power of telescopes is the **illumination J**, the amount of light energy per second focused onto a unit area of the resolved image.

$$J \propto D^2/f^2$$

define **focal ratio**:

$$F \equiv \frac{f}{D}$$

Since the number of photons per second striking a unit area of photographic plate or some other detector is described by the illumination, the illumination indicates the amount of time required to collect the photons needed to form a sufficiently bright image for analysis.

We now see that the size of the aperture of a telescope is critical for two reasons: A larger aperture both improves resolution and increases the illumination. On the other hand, a longer focal length increases the linear size of the image but decreases the illumination. For a fixed focal ratio, increasing the diameter of the telescope results in greater spatial resolution, but the illumination remains constant. The proper design of a telescope must take into account the principal applications that are intended for the instrument.

6.2 OPTICAL TELESCOPES

6.2.1 Refracting Telescopes

The difficulties with a large objective lens.
Considering all of the challenges inherent in the design and construction of the refracting telescopes, the vast majority of all large modern telescopes are reflecting telescopes.

6.2.2 Reflecting Telescopes

In fact, because the mirror is supported from behind rather than along its edges, it is possible to design an active system of pressure pads that can help to eliminate distortions in the mirror's shape produced by thermal effects and the changes in the gravitational force on the mirror as the telescope moves (a process known as **active optics**).

6.2.3 Telescope Mounts

equatorial mount

6.2.4 Large-Aperture Telescopes

6.2.5 Adaptive Optics

Fluctuations in the guide star determine the adjustments that must be made to the deformable mirror.

6.2.6 Space-Based Observatories

6.2.7 Electronic Detectors

6.3 RADIO TELESCOPES

Today radio astronomy plays an important role in our investigation of the electromagnetic spectrum. Radio waves are produced by a variety of mechanisms related to a range of physical processes, such as the interactions of charged particles with magnetic fields. This window on the universe provides astronomers and physicists with valuable clues to the inner workings of some of nature's most spectacular phenomena.

6.3.1 Spectral Flux Density

Since radio waves interact with matter differently than visible light does, the devices used to detect and measure it are necessarily very different from optical telescopes. The parabolic dish of a typical radio telescope reflects the radio energy of the source to an antenna. The signal is then amplified and processed to produce a radio map of the sky at a particular wavelength.
The strength of a radio source is measured in terms of the **spectral flux density**, $S(\nu)$, the amount of energy per second, per unit frequency interval striking a unit area of the telescope. To determine the total amount of energy per second (the power) collected by the receiver, the spectral flux must be integrated over the telescope's collecting area and over the frequency interval for which the detector is sensitive, referred to as the *bandwidth.* If f_ν, is a function describing the efficiency of the detector at the frequency ν, then the amount of energy detected per second becomes:

$$P = \int_A \int_v S(v) f_v dv dA$$

(A similar formula applies to optical telescopes since filters and detectors (including the human eye) are frequency dependent.)

6.3.2 Improving Resolution: Large Apertures and Interferometry

Clearly the ability to resolve an image improves with a longer baseline d. **Very long baseline interferometry** (VLBI) is possible over the size of a continent or even between continents. In such cases the data can be recorded on site and delivered to a central location for processing at a later time. It is only necessary that the observations be simultaneous and that the exact time of data acquisition be recorded.

antenna pattern
The narrowness of the main lobe is described by specifying its angular width at half its length, referred to as the **half-power beam width** (HPBW). this width can be decreased, and the effect of the side lobes can be significantly reduced, by the addition of other telescopes to produce the desired diffraction pattern. This property is analogous to the increase in sharpness of a grating diffraction pattern as the number of grating lines is increased.

At those wavelengths ALMA will be able to probe deeply into dusty regions of space where stars and planets are believed to be forming, as well as to study the earliest stages of galaxy formation—all critical problems in modern astrophysics.

6.4 INFRARED, ULTRAVIOLET, X-RAY, AND GAMMA-RAY ASTRONOMY

6.4.1 Atmospheric Windows in the Electromagnetic Spectrum

The primary contributor to infrared absorption is water vapor.

6.4.2 Observing Above the Atmosphere

The data from these telescopes have given astronomers important information concerning a vast array of astrophysical processes, including mass loss from hot stars, cataclysmic variable stars, and compact objects such as white dwarfs and pulsars.

At even shorter wavelengths, X-ray and gamma-ray astronomy yields information about very energetic phenomena, such as nuclear reaction processes and the environments around black holes. As a result of the very high photon energies involved, X-ray and gamma-ray observations require techniques that differ markedly from those at longer wavelengths. For instance, traditional glass mirrors are useless for forming images in this regime because of the great penetrating power of these photons. However, it is still possible to image sources by using grazing-incidence reflections (incident angle close to 90°). X-ray spectra can also be obtained using techniques such as Bragg scattering, an interference phenomenon produced by photon reflections from atoms in a regular crystal lattice. The distance between the atoms corresponds to the separation between slits in an optical diffraction grating.

In 1970 UHURU (also known as the Small Astronomy Satellite—1, SAS 1) made the first comprehensive survey of the X-ray sky. In the late 1970s the three High Energy Astrophysical Observatories, including the Einstein Observatory, discovered thousands of X-ray and gamma-ray sources. Between 1990 and 1999, the X-ray observatory ROSAT (the Roentgen Satellite), a German—American—British satellite consisting of two detectors and an imaging telescope operating in the range of 0.51 nm to 12.4 nm, investigated the hot coronas of stars, supernova remnants, and quasars. Japan's Advanced Satellite for Cosmology and Astrophysics, which began its mission in 1993, also made valuable X-ray observations of the heavens before attitude control was lost as a result of a geomagnetic storm July 14, 2000.

Launched in 1999 and named for the Nobel Prize-winning astrophysicist Subrahmanyan Chandrasekhar (1910-1995), the Chandra X-ray Observatory operates in the energy range from 0.2 keV to 10 keV (6.2 nm to 0.1 nm, respectively) with an angular resolution of approximately 0.5". Because X-rays cannot be focused in the same way that longer wavelengths can, grazing incidence mirrors are used to achieve the outstanding resolving power of Chandra.
The European Space Agency operates another X-ray telescope also launched in 1999, the X-ray Multi-Mirror Newton Observatory (XMM-Newton). Complementing Chandra's sensitivity range, XMM-Newton operates between 0.01 nm and 1.2 nm.
The Compton Gamma Ray Observatory [CGRO] observed the heavens at wavelengths shorter than those measured by the X-ray telescopes. Placed into orbit by the Space Shuttle Atlantis 1991, the observatory was deorbited into the Pacific Ocean in June 2000.

6.5 ALL-SKY SURVEYS AND VIRTUAL OBSERVATORIES

Moreover, from the ground, large-scale, automated surveys have been, or are being, conducted in various wavelength regimes. For instance, the visible Sloan Digital Sk Survey (SDSS) and the near infrared Two-Micron All Sky Survey (2MASS) will produc tremendous volumes of data that need to be analyzed. SDSS alone will result in 15 terabyte: of data (comparable to all of the information contained in the Library of Congress). Petabyte sized data sets are also being envisioned in the not-too-distant future.

Given the enormous volumes of data that have already been and will be produced by ground-based and space-based astronomical observatories, together with the tremendous amount of information that already exists in on-line journals and databases, great attention is being given to developing **web-based virtual observatories**. The goal of these projects is to create user interfaces that give astronomers access to already-existing observational data. For instance, an astronomer could query a virtual observatory database for all of the observations that have ever been made in a specified region of the sky over any wavelength band. That data would then be downloaded to the astronomer's desktop computer or mainframe for study. In order to accomplish this task, common data formats must be created, and data analysis and visualization tools must be developed to aide in this challenging project in information technology. At the time this text was written, several prototype virtual observatories had been developed, such as Skyview hosted by NASA's Goddard Space Flight Center, or the Guide Star Catalogs and the Digitized Sky Survey maintained by the Space Telescope Science Institute. On-line access to a large number of currently existing databases are also available at the National Space Science Data Center (NSSDC). In addi- tion, several initiatives are under way to integrate and standardize the efforts. In the United States, the National Science Foundation (NSF) has funded the National Virtual Observatory project; in Europe, the Astrophysical Virtual Observatory project is under way; the United Kingdom is pursuing Astrogrid; and Australia is working on the Australian Virtual Observatory. It is hoped that all of these efforts will ultimately be combined to create an International Virtual Observatory.

With the past successes of ground-based and orbital observatories, astronomers have been able to make great strides in our understanding of the universe. Given the current advances in detectors, observational techniques, new observational facilities, and virtual observatories, the future holds tremendous promise for providing significantly improved studies of known objects in the heavens. However, perhaps the most exciting implications of these observational advances are to be found in as yet undiscovered and unanticipated phenomena.

SUGGESTED READING

Telescopes - PROBLEM SET

Chapter 7

Binary Systems and Stellar Parameters

7.1 THE CLASSIFICATION OF BINARY STARS

A detailed understanding of the structure and evolution of stars requires knowledge of their physical characteristics. We have seen that knowledge of blackbody radiation curves, spectra, and parallax enables us to determine a star's effective temperature, luminosity, radius, composition, and other parameters. However, the only direct way to determine the mass of a star is by studying its gravitational interaction with other objects. Kepler's laws can be used to calculate the masses of members of our Solar System. However, the universality of the gravitational force allows Kepler's laws to be generalized to include the orbits of stars about one another and even the orbital interactions of galaxies, as long as proper care is taken to refer all orbits to the center of mass of the system.

Binary star systems are classified according to their specific observational characteristics:
Optical double, Visual binary, Astrometric binary, Eclipsing binary, Spectrum binary(Doppler effect, red/blue shift), Spectroscopic binary(spectral lines).

Three types of systems can provide us with mass determinations: visual binaries combined with parallax information; visual binaries for which radial velocities are available over a complete orbit; and eclipsing, double-line, spectroscopic binaries.

7.2 MASS DETERMINATION USING VISUAL BINARIES

When the angular separation between components of a binary system is greater than the resolution limit imposed by local seeing conditions and the fundamental diffraction limitation of the Rayleigh criterion, it becomes possible to analyze the orbital characteristics of the individual stars. From the orbital data, the orientation of the orbits and the system's center of mass can be determined, providing knowledge of the ratio of the stars' masses. If the distance to the system is also known, from trigonometric parallax for instance, the linear separation of the stars can be determined, leading to the individual masses of the stars in the system.

7.3 ECLIPSING, SPECTROSCOPIC BINARIES

A wealth of information is available from a binary system even if it is not possible to resolve each of its stars individually. This is particularly true for a double-line, eclipsing, spectroscopic binary star system. In such a system, not only is it possible to determine the individual masses of the stars, but astronomers may be able to deduce other parameters as well, such as the stars' radii and the ratio of their fluxes, and hence the ratio of their effective temperatures. (Of course, eclipsing systems are not restricted to spectroscopic binaries but may occur in other types of binaries as well, such as visual binaries.)

7.3.1 The Effect of Eccentricity on Radial Velocity Measurements

7.3.2 The Mass Function and the Mass–Luminosity Relation

Evaluating masses of binaries has shown the existence of a well-defined **mass—luminosity relation** for the large majority of stars in the sky. One of the goals of the next several chapters is to investigate the origin of this relation in terms of fundamental physical principles.

7.3.3 Using Eclipses to Determine Radii and Ratios of Temperatures

The ratio of the effective temperatures of the two stars can also be obtained from the light curve of an eclipsing binary. This is accomplished by considering the objects as blackbody radiators and comparing the amount of light received during an eclipse with the amount received when both members are fully visible.

7.3.4 A Computer Modeling Approach

The modern approach to analyzing the data from binary star systems involves computing detailed models that can yield important information about a variety of physical parameters. Not only can masses, radii, and effective temperatures be determined, but for many systems other details can be described as well. For instance, gravitational forces, combined with the effects of rotation and orbital motion, alter the stars' shapes; they are no longer simply spherical objects but may become elongated.

The study of binary star systems provides valuable information about the observable characteristics of stars. These results are then employed in developing a theory of stellar structure and evolution.

7.4 THE SEARCH FOR EXTRASOLAR PLANETS

For hundreds of years, people have looked up at the night sky and wondered if planets might exist around other stars. However, it wasn't until October 1995 that Michel Mayor and Didier Queloz of the Geneva Observatory announced the discovery of a planet around the solar-type star 51 Pegasi.

This modern discovery of extrasolar planets at such a prodigious rate was made possible by dramatic advances in detector technology, the availability of large-aperture telescopes, and diligent, long-term observing campaigns. Given the huge disparity between the luminosity of the parent star and any orbiting planets, direct observation of a planet has proved very difficult; the planet's reflected light is simply overwhelmed by the luminosity of the star. As a result, more indirect methods are usually required to detect extrasolar planets! Three techniques that have all been used successfully are based on ideas discussed in this chapter: radial velocity measurements, astrometric wobbles, and eclipses. The first method, the detection of radial velocity variations in parent stars induced by the gravitational tug of the orbiting planets has been by far the most prolific method at the time this text was written!(现在, 2019年, transit方法产出最多!)

In 1992, Alexander Wolszczan, of the Arecibo Radio Observatory in Puerto Rico, and Dale Frail, of the National Radio Astronomy Observatory, detected three

Earth- and Moon-sized planets around a pulsar (PSR 1257+12), an extremely compact collapsed star that was produced following a supernova explosion. **This discovery was made by noting variations in the extremely regular radio emission coming from the collapsed star.**
In April 2004, G. Chauvin and colleagues used the VLT/NACO of the European Southern Observatory to obtain an infrared image of a giant extrasolar planet of spectral type between LS and L9.5 orbiting the brown dwarf 2MASSWI1207334-393254. HST/NICMOS was also able to observe the brown dwarf's planetary companion. 8 Another technique has also been employed in the search for extrasolar planets; it is based on the gravitational lensing of light.

Incredibly, today it is possible to measure radial velocity variations as small as $3ms^{-1}$!, a slow jog in the park. Marcy, Butler, and their research-team colleagues accomplish this level of detection by passing starlight through an iodine vapor. The imprinted absorption lines from the iodine are used as zero-velocity reference lines in the high-resolution spectrum of the star. By comparing the absorption and emission-line wavelengths of the star to the iodine reference wavelengths, it is possible to determine very precise radial velocities. The high-resolution spectrographs used by the team were designed and built by another team member, Steve Vogt of the University of California, Santa Cruz.

The analysis of the radial velocities requires much more work before the true reflex motion radial velocity variations of the star can be deduced, however. In order to determine the source of the variations, it is first necessary to eliminate all other sources of radial velocities superimposed on the observed spectra. These include the rotation and wobble of Earth, the orbital velocity of Earth around the Sun, and the gravitational effects of the other planets in our Solar System on Earth and our Sun. After all of these corrections have been made, the radial velocity of the target star can be referenced to the true center of mass of our Solar System.

In addition to the motions in our Solar System, motions of the target star itself must be taken into account. For instance, if the target star is rotating, radial velocities due to the approaching and receding edges of its apparent disk will blur the absorption lines used to measure radial velocity. Pulsations of the surface of the star, surface convection, and the movement of surface features such as star spots, can also con- fuse the measurements and degrade the velocity resolution limit. All of the planets discovered by the radial velocity technique are quite close to their parent star and very massive.

SUGGESTED READING

Binary Systems and Stellar Parameters - PROBLEM SET

Chapter 8

The Classification of Stellar Spectra

8.1 THE FORMATION OF SPECTRAL LINES

With the invention of photometry and spectroscopy, the new science of astrophysics pro- gressed rapidly. As early as 1817, Joseph Fraunhofer had determined that different stars have different spectra. Stellar spectra were classified according to several schemes, the ear- liest of which recognized just three types of spectra. As instruments improved, increasingly subtle distinctions became possible.

8.1.1 The Spectral Type of Stars

With these changes, the Harvard classification scheme of "O BAF GK M" became a temperature sequence, running from the hottest blue O stars to the coolest red M stars. Generations of astronomy students have remembered this string of **spectral types** by memorizing the phrase "Oh Be A Fine Girl/Guy, Kiss Me." Stars nearer the beginning of this sequence are referred to as early-type stars, and those closer to the end are called late-type stars. These labels also distinguish the stars within the spectral subdivisions, so astronomers may speak of a KO star as an "early K star" or refer to a BO star as a "late B star." Cannon classified some 200,000 spectra between 1911 and 1914, and the results were collected into the **Henry Draper Catalogue**. Today, many stars are referred to by their HD numbers; Betelgeuse is HD 39801.

The physical basis of the Harvard spectral classification scheme remained obscure, however. **The theoretical understanding of the quantum atom achieved early in the twentieth century gave astronomers the key to the secrets**

of stellar spectra

Absorption lines are created when an atom absorbs a photon with exactly the energy required for an electron to make an upward transition from a lower to a higher orbital. Emission lines are formed in the inverse process, when an electron makes a downward transition from a higher to a lower orbital and a single photon carries away the energy lost by the electron. The wavelength of the photon thus depends on the energies of the atomic orbitals involved in these transitions.

The distinctions between the spectra of stars with different temperatures are due to electrons occupying different atomic orbitals in the atmospheres of these stars. The details of spectral line formation can be quite complicated because electrons can be found in any of an atom's orbitals. Furthermore, the atom can be in any one of various stages of ionization and has a unique set of orbitals at each stage. An atom's stage of ionization is denoted by a Roman numeral following the symbol for the atom. For example, H I and He I are neutral (not ionized) hydrogen and helium, respectively; He II is singly ionized helium, and Si III and Si IV refer to a silicon atom that has lost two and three electrons, respectively.

In the table the term **metal** is used to indicate any element heavier than helium, a convention commonly adopted by astronomers because by far the most abundant elements in the universe are hydrogen and helium.

Brown dwarfs are objects with too little mass to allow nuclear reactions to occur in their interiors in any substantial way, so they are not considered stars in the usual sense.

Readily apparent is the shifting to longer wavelengths of the peak of the superimposed blackbody spectrum as the temperature of the star decreases (later spectral types).
Note how these hydrogen absorption lines grow in strength from O to A and then decrease in strength for spectral types later than A. For later spectral types, the messy spectra are indicative of metal lines, with molecular lines appearing in the spectra of the coolest stars.

8.1.2 The Maxwell–Boltzmann Velocity Distribution

To uncover the physical foundation of this classification system, two basic questions must be answered: In what orbitals are electrons most likely to be found? What are the relative numbers of atoms in various stages of ionization?

The answers to both questions are found in an area of physics known as **statistical mechanics**. This branch of physics studies the statistical properties of a system composed of many members. For example, a gas can contain a huge number of particles with a large range of speeds and energies. Although in practice it would be impossible to calculate the detailed behavior of any single particle, the gas as a whole does have certain well defined properties, such as its temperature, pressure, and density. For such a gas in thermal equilibrium (the gas is not rapidly increasing or decreasing in temperature, for instance), the **Maxwell–Boltzmann velocity distribution function** describes the fraction of particles having a given range of speeds. The number of gas particles per unit volume having speeds between v and $v + dv$ is given by:

$$n_v dv = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv$$

where n is the total number density (number of particles per unit volume), $n_v \equiv \partial n/\partial v$, m is a particle's mass, k is Boltzmann's constant, and T is the temperature of the gas in kelvins.

The exponent of the distribution function is the ratio of a gas particle's kinetic energy, $\frac{1}{2}mv^2$, to the characteristic thermal energy, kT. It is difficult for a significant number of particles to have an energy much greater or less than the thermal energy; the distribution peaks when these energies are equal, at a most probable speed of:

$$v_{\rm mp} = \sqrt{\frac{2kT}{m}}$$

The high-speed exponential "tail" of the distribution function results in a somewhat higher (average) **root-mean-square speed** of:

$$v_{\rm rms} = \sqrt{\frac{3kT}{m}}$$

8.1.3 The Boltzmann Equation

The atoms of a gas gain will lose energy as they collide. As a result, the distribution in the speeds of the impacting atoms, produces a definite distribution of the electrons among the atomic orbitals. **This distribution of electrons is governed by a fundamental result of statistical mechanics: Orbitals of higher energy are less likely to be occupied by electrons.**

It is often the case that the energy levels of the system may be **degenerate**, with more than one quantum state having the same energy.

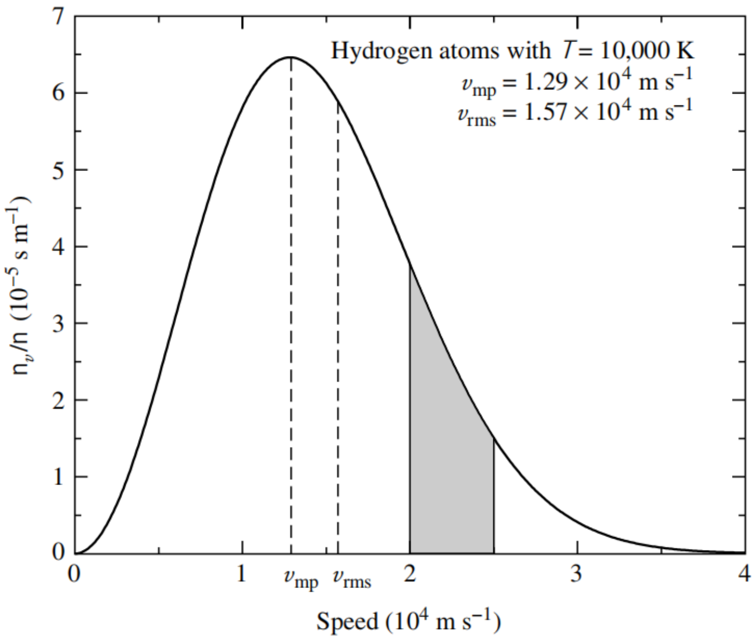

Figure 8.1: Maxwell–Boltzmann distribution function,n_v/n, for hydrogen atoms at a temperature of 10,000 K. The fraction of hydrogen atoms in the gas having velocities between 2×10^4 ms^{-1} and 2.5×10^4 ms^{-1} is the shaded area under the curve between those two velocities.

For the atoms of a given element in a specified state of ionization, the ratio of the number of atoms N_b with energy E_b to, for the atoms of a given element in a specified state of ionization, the ratio of the number of atoms N_b with energy E_b to the number of atoms N_a with energy E_a in different states of excitation is given by the **Boltzmann equation:**

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b-E_a)/kT}$$

the degeneracy of the nth energy level of the hydrogen atom is $g_n = 2n^2$

Example 1.3:
High temperatures are required for a significant number of hydrogen atoms to have electrons in the first excited state. Figure 7 shows the relative occupancy of the ground and first excited states, $N_2/(N_1 + N_2)$, as a function of temperature. This result is somewhat puzzling, however. Recall that **the Balmer absorption lines are produced by electrons in hydrogen atoms making an upward transition from the $n = 2$ orbital. If, as shown in Example 1.3, temperatures**

on the order of 85,000 K are needed to provide electrons in the first excited state, then why do the Balmer lines reach their maximum intensity at a much lower temperature of 9520 K? Clearly, **according to Boltzmann equation, at temperatures higher than 9520 K an even greater proportion of the electrons will be in the first excited state rather than in the ground state. If this is the case, then what is responsible for the diminishing strength of the Balmer lines at higher temperatures?**
(这段的意思是:
1.根据波尔兹曼方程算出来的结果表明, 需要85000K才能提供足够的(50%)处于第一激发态的电子, 巴尔末吸收线产生于氢原子中第一激发态的电子吸收能量能级向上迁移, 但这一迁移在 9250K 最强;
2.从另一角度上来看, 9250K 都怎么强, 根据波尔兹曼方程, 在更高的温度下, 处于第一激发态的电子更多(理应有更强的巴尔末线), 是什么造成了高温端的巴尔末线强度减弱呢?)

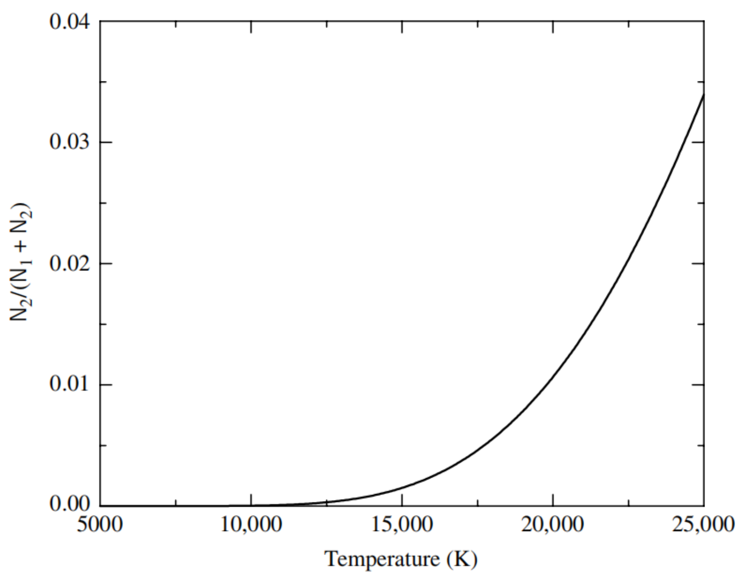

Figure 8.2: $N_2/(N_1+N_2)$ for the hydrogen atom obtained via the Boltzmann equation.

8.1.4 The Saha Equation

The answer lies in also considering the relative number of atoms in different stages of ionization.

Let χ_i be the ionization energy needed to remove an electron from an atom (or ion) in the ground state, thus taking it from ionization stage i to stage (i + 1). However, it may be that the initial and final ions are not in the ground state. An average must be taken over the orbital energies to allow for the possible partitioning of the atom's electrons among its orbitals. This procedure involves calculating the **partition functions**, Z, for the initial and final atoms. The partition function is simply the weighted sum of the number of ways the atom can arrange its electrons with the same energy, with more energetic (and therefore less likely) configurations receiving less weight from the Boltzmann factor when the sum is taken. If E_j is the energy of the j th energy level and g_j is the degeneracy of that level, then the partition function Z is defined as:

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT}$$

If we use the partition functions Z_i and Z_{i+1} for the atom in its initial and final stages of ionization, the ratio of the number of atoms in stage (i + 1) to the number of atoms in stage i is:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

This equation is known as the **Saha Equation**, after the Indian astrophysicist Meghnad Saha (1894–1956), who first derived it in 1920.

Because a free electron is produced in the ionization process, it is not surprising to find the number density of free electrons (number of free electrons per unit volume), n_e, on the right-hand side of the Saha equation. Note that as the number density of free electrons increases, the number of atoms in the higher stage of ionization decreases, since there are more electrons with which the ion may recombine.
The factor of 2 in front of the partition function Z_{i+1} reflects the two possible spins of the free electron, with $m_s = \pm 1/2$.
The term in parentheses is also related to the free electron, with m_e being the electron mass. (The term in parentheses is the number density of electrons for which the quantum energy is roughly equal to the characteristic thermal energy kT . For the classical conditions encountered in stellar atmospheres, this term is much greater than n_e.)

8.1.5 Combining the Boltzmann and Saha Equations

We are now finally ready to consider the combined effects of the Boltzmann and Saha equations and how they influence the stellar spectra that we observe.

Example 1.4(重点,需要重看)
Figure 9 shows that in this example, the hydrogen gas would produce the most intense Balmer lines at a temperature of 9900 K, in good agreement with the observations. The diminishing strength of the Balmer lines at higher temperatures is due to the rapid ionization of hydrogen above 10,000 K.
Figure 8.3 summarizes this situation. Of course, stellar atmospheres are not com-

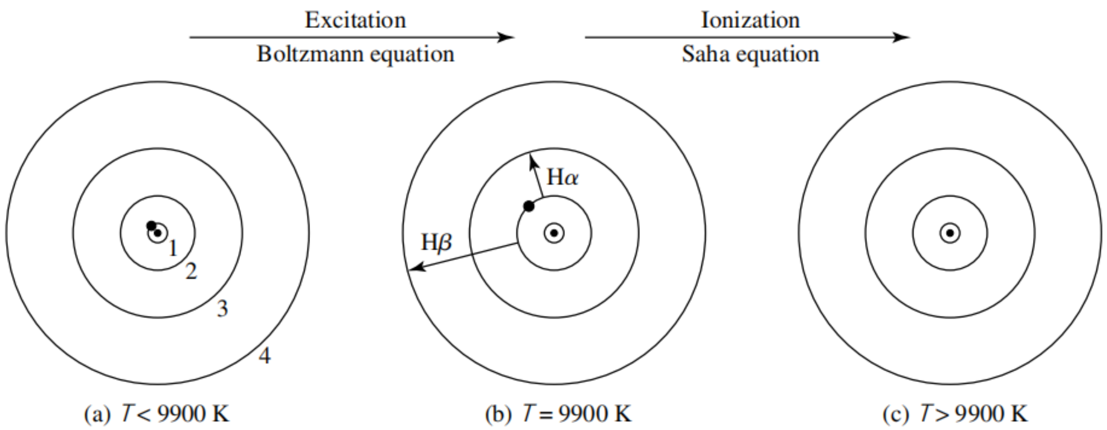

Figure 8.3: The electron's position in the hydrogen atom at different temperatures. In (a), the electron is in the ground state. Balmer absorption lines are produced only when the electron is initially in the first excited state, as shown in (b). In (c), the atom has been ionized.

posed of pure hydrogen, and the results obtained in Example 1.4 depended on an appropriate value for the electron pressure. In stellar atmospheres, there is typically one helium atom for every ten hydrogen atoms. The presence of ionized helium provides more electrons with which the hydrogen ions can recombine. Thus, when helium is added, it takes a higher temperature to achieve the same degree of hydrogen ionization.

It should also be emphasized that the Saha equation can be applied only to a gas in thermodynamic equilibrium, so that the Maxwell–Boltzmann velocity distribution is obeyed. Furthermore, the density of the gas must not be too great (less than roughly 1 $kg\ m^{-3}$ for stellar material), or the presence of neighboring ions will distort an atom's orbitals and lower its ionization energy.

Example 1.5:
Use the Saha equation to determine the degree of ionization and will use the Boltzmann equation to reveal the distribution of electrons between the ground and first

excited states.

Figure 11 shows how the strength of various spectral lines varies with spectral type and temperature. As the temperature changes, a smooth variation from one spectral type to the next occurs, indicating that there are only minor differences in the composition of stars, as inferred from their spectra. The first person to determine the composition of the stars and discover the dominant role of hydrogen in the universe was Cecilia Payne (1900–1979). **Her 1925 Ph.D. thesis, in which she calculated the relative abundances of 18 elements in stellar atmospheres, is among the most brilliant ever done in astronomy**

8.2 THE HERTZSPRUNG–RUSSELL DIAGRAM

Early in the twentieth century, as astronomers accumulated data for an increasingly large sample of stars, they became aware of the wide range of stellar luminosities and absolute magnitudes.

These regularities led to a theory of stellar evolution16 that described how stars might cool off as they age. This theory (no longer accepted) held that stars begin their lives as young, hot, bright blue O stars. It was suggested that as they age, stars become less massive as they exhaust more and more of their "fuel" and that they then gradually become cooler and fainter until they fade away as old, dim red M stars. Although incorrect, a vestige of this idea remains in the terms early and late spectral types

8.2.1 An Enormous Range in Stellar Radii

If this idea of stellar cooling were correct, then there should be a relation between a star's absolute magnitude and its spectral type.

The radius of a star can be easily determined from its position on the H–R diagram.

The existence of such a simple relation between luminosity and temperature for mainsequence stars is a valuable clue that the position of a star on the main sequence is governed by a single factor. This factor is the star's mass.

Combining the radii and masses known for main-sequence stars, we can calculate the average density of the stars. The result, perhaps surprising, is that main-sequence stars have roughly the same density as water. Moving up the main sequence, we find that the larger, more massive, early-type stars have a lower average density(一方面, 核聚变不断产生更重的物质, 另一方面, 核燃烧之后恒星不断塌缩)

8.2.2 Morgan–Keenan Luminosity Classes

The two-dimensional M–K classification scheme enables astronomers to locate a star's position on the Hertzsprung–Russell diagram based entirely on the appearance of its spectrum. Once the star's absolute magnitude, M, has been read from the vertical axis of the H–R diagram, the distance to the star can be calculated from its apparent magnitude, m:

$$d = 10^{(m-M+5)/5}$$

where d is in units of parsecs. This method of distance determination, called spectroscopic parallax, is responsible for many of the distances measured for stars, but its accuracy is limited because there is not a perfect correlation between stellar absolute magnitudes and luminosity classes.

SUGGESTED READING

The Classification of Stellar Spectra - PROBLEM SET

Chapter 9

Stellar Atmospheres

9.1 THE DESCRIPTION OF THE RADIATION FIELD

The light that astronomers receive from a star comes from the star's atmosphere, the layers of gas overlying the opaque interior. A flood of photons pours from these layers, releasing the energy produced by the thermonuclear reactions, gravitational contraction, and cooling in the star's center. The temperature, density, and composition of the atmospheric layers from which these photons escape determine the features of the star's spectrum. To interpret the observed spectral lines properly, we must describe how light travels through the gas that makes up a star.

9.1.1 The Specific and Mean Intensities

9.1.2 The Specific Energy Density

9.1.3 The Specific Radiative Flux

Both the radiative flux and the specific intensity describe the light received from a celestial source, and you may wonder which of these quantities is actually measured by a telescope's photometer, pointed at the source of light. The answer depends on whether the source is resolved by the telescope.

However, it is the radiative flux that is measured for an unresolved source. As the source recedes farther and farther, it will eventually subtend an angle θ smaller than θ_{min}, and it can no longer be resolved by the telescope. When $\theta < \theta_{min}$, the energy received from the entire source will disperse throughout the diffraction pattern (the Airy disk and rings) determined by the telescope's aperture. Because the light arriving at the detector leaves the surface of the source at all angles [see Fig. 3(b)], the detector is effectively integrating the specific intensity over all directions. This is just the definition of the radiative flux.

9.1.4 Radiation Pressure

Because a photon possesses an energy E, Einstein's relativistic energy equation tells us that even though it is massless, a photon also carries a momentum of $p = E/c$ and thus can exert a radiation pressure. This radiation pressure can be derived in the same way that gas pressure is found for molecules bouncing off a wall.

Thus the blackbody radiation pressure is one-third of the energy density. (For comparison, the pressure of an ideal monatomic gas is two-thirds of its energy density.)

9.2 STELLAR OPACITY

The classification of stellar spectra is an ongoing process. Even the most basic task, such as finding the "surface" temperature of a particular star, is complicated by the fact that stars are not actually blackbodies. The Stefan–Boltzmann relation defines a star's effective temperature, but some effort is required to obtain a more accurate value of the "surface" temperature.
Figure 9.1 shows that the Sun's spectrum deviates substantial from the shape of the blackbody Planck function, B;, because solar absorption lines remove light from the Sun's continuous spectrum at certain wavelengths. The decrease in intensity produced by the dense series of metallic absorption lines in the solar spectrum is especially effective; this effect is called **line blanketing**. In other wavelength regimes (e.g., X-ray an UV), emission lines may augment the intensity of the continuous spectrum.

9.2.1 Temperature and Local Thermodynamic Equilibrium

Although we often think in terms of the temperature at a particular location, there are actually many different measures of temperature within a star, defined according to the physical process being described:

- The **effective temperature**, which is obtained from the Stefan–Boltzmann law is uniquely defined for a specific level within a star and is an important global descriptor of that star.

- The **excitation temperature** is defined by the Boltzmann equation.

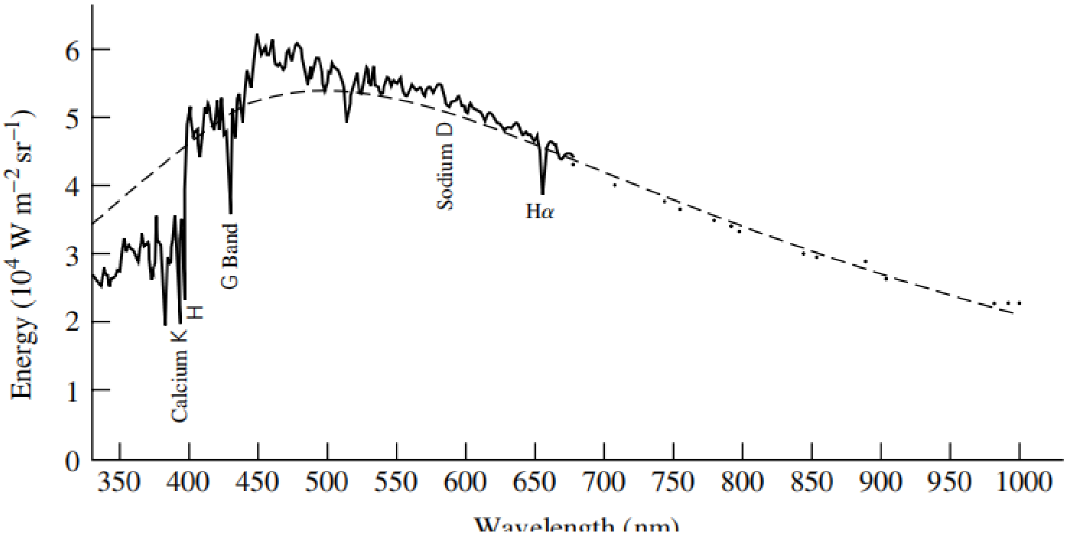

Figure 9.1: The spectrum of the Sun in 2 nm wavelength intervals. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure adapted from Aller, Atoms, Stars, and Nebulae, Third Edition, Cambridge University Press, New York, 1991.)

- The **ionization temperature** is defined by the Saha equation
- The **kinetic temperature** is contained in the Maxwell–Boltzmann distribution
- The **color temperature** is obtained by fitting the shape of a star's continuous spectrum to the Planck function

In such a steady-state condition, no net flow of energy through the box or between the matter and the radiation occurs. Every process (e.g., the absorption of a photon) occurs at the same rate as its inverse process (e.g., the emission of a photon). This condition is called **thermodynamic equilibrium**.

However, a star cannot be in perfect thermodynamic equilibrium. A net outward flow of energy occurs through the star, and the temperature, however it is defined, varies with location. Gas particles and photons at one position in the star may have arrived there from other regions, either hotter or cooler (in other words, there is no "ideal box"). The distribution in particle speeds and photon energies thus reflects a range of temperatures. As the gas particles collide with one another and interact with the radiation field by absorbing and emitting photons, the description of the processes of excitation and ionization becomes quite complex. However, the idealized case of a single temperature can still be employed if the distance over which the temperature changes significantly is large compared with the distances traveled by the particles and photons between collisions (their mean free paths). In this case, referred to as **local thermodynamic equilibrium** (LTE), the particles and photons cannot escape the local environment and so are effectively confined to a limited volume (an approximated "box") of nearly constant temperature.

The concept of cross section actually represents a probability of particle interactions but has units of cross sectional area.

mean free path between collisions

9.2.2 The Definition of Opacity

We now turn to a consideration of a beam of parallel light rays traveling through a gas. Any process that removes photons from a beam of light will be collectively termed **absorption**. In this sense then, absorption includes the scattering of photons (such as Compton scattering) as well as the true absorption of photons by atomic electrons making upward transitions. In sufficiently cool gases, molecular energy-level transitions may also occur and must be included.

The change in the intensity, dI_λ, of a ray of wavelength λ as it travels through a gas is proportional to its intensity, I_λ, the distance traveled, ds, and the density of the gas, ρ. That is,

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$$

The distance s is measured along the path traveled by the beam and increases in the direction that the beam travels; the minus sign in Eq. (13) shows that the intensity decreases with distance due to the absorption of photons. The quantity κ_λ is called the absorption coefficient, or opacity, with the λ subscript implicitly indicating that the opacity is wavelength dependent (κ_λ is sometimes referred to as a monochromatic opacity). The opacity is the cross section for absorbing photons of wavelength λ per unit mass of stellar material and has units of m^2 kg^{-1}. In general, the opacity of a gas is a function of its composition, density, and temperature.

9.2.3 Optical Depth

If $\tau_\lambda \gg 1$ for a light ray passing through a volume of gas, the gas is said to be optically thick; if $\tau_\lambda \ll 1$, the gas is optically thin. Because the optical depth varies with wavelength, a gas may be optically thick at one wavelength and optically thin at another. For example, Earth's atmosphere is optically thin at visible wavelengths (we can see the stars), but optically thick at X-ray wavelengths.

9.2.4 General Sources of Opacity

The opacity of the stellar material is determined by the details of how photons interact with particles (atoms, ions, and free electrons). If the photon passes within σ_λ of the particle, where σ_λ is the particle's cross-sectional area (or effective target area), the photon may be either absorbed or scattered. In an absorption process, the photon ceases to exist and its energy is given up to the thermal energy of the gas. In a scattering process the photon continues on in a different direction. Both absorption and scattering can remove photons from a beam of light, and so contribute to the opacity, κ_λ, of the stellar material. If the opacity varies slowly with wavelength, it determines the star's continuous spectrum (or continuum). The dark absorption lines superimposed on the continuum are the result of a rapid variation in the opacity with wavelength.

In general, there are four primary sources of opacity available for removing stellar photons from a beam. Each involves a change in the quantum state of an electron, and the terms bound and free are used to describe whether the electron is bound to an atom or ion in its initial and final states:

- Bound–bound transitions (excitations and de-excitations) occur when an electron in an atom or ion makes a transition from one orbital to another. Thus $\kappa_{\lambda,bb}$, the bound–bound opacity, is small except at those discrete wavelengths capable of producing an upward atomic transition. It is $\kappa_{\lambda,bb}$ that is responsible for forming the absorption lines in stellar spectra.

  The net result of this absorption–emission sequence is essentially a scattered photon. Otherwise, if the electron makes a transition to an orbital other than its initial one, the original photon is not recovered and the process is one of true absorption. If, while in its excited state, the atom or ion collides with a neighboring particle, collisional de-excitation may result. When this occurs, the energy lost by the atom or ion becomes a part of the thermal energy of the gas.

  An important by-product of this absorption process is degrading of the average energy of the photons in the radiation field. For example, if one photon is absorbed but two photons are emitted as the electron cascades down to its initial orbital, then the average photon energy has been reduced by half.

There is no simple equation for bound–bound transitions that describes all of the contributions to the opacity by individual spectral lines.

- Bound–free absorption, also known as photoionization, occurs when an incident photon has enough energy to ionize an atom.
  Thus $\kappa_{\lambda,bf}$, the bound–free opacity, is one source of the continuum opacity.

  As with bound–bound emission, this also contributes to reducing the average energy of the photons in the radiation field.

- Free–free absorption is a scattering process, shown in Fig. 9, that takes place when a free electron in the vicinity of an ion absorbs a photon, causing the speed of the electron to increase. In this process the nearby ion is necessary in order to conserve both energy and momentum.
  Since this mechanism can occur for a continuous range of wavelengths, free–free opacity, $\kappa_{\lambda,ff}$, is another contributor to the continuum opacity. It may also happen that as it passes near an ion, the electron loses energy by emitting a photon, which causes the electron to slow down. This process of free–free emission is also known as **bremsstrahlung**, which means "braking radiation" in German.

- Electron scattering is as advertised: A photon is scattered (not absorbed) by a free electron through the process of Thomson scattering. In this process, the electron can be thought of as being made to oscillate in the electromagnetic field of the photon. However, because the electron is tiny, it makes a poor target for an incident photon, resulting in a small cross section. The cross section for Thomson scattering has the same value for photons of all wavelengths.
  The small size of the Thomson cross section means that electron scattering is most effective as a source of opacity when the electron density is very high, which requires high temperature. In the atmospheres of the hottest stars (and in the interiors of all stars), where most of the gas is completely ionized, other sources of opacity that involve bound electrons are eliminated. In this high-temperature regime, the opacity due to electron scattering, κ_{es}, dominates the continuum opacity.

A photon may also be scattered by an electron that is loosely bound to an atomic nucleus. This result is called Compton scattering if the photon's wavelength is much smaller than the atom or Rayleigh scattering if the photon's wavelength is much larger. In Compton scattering, the change in the wavelength and energy of the scattered photon is very small, so Compton scattering is usually lumped together with Thomson scattering.

The cross section for Rayleigh scattering from a loosely bound electron is smaller than the Thomson cross section; it is proportional to $1/\lambda^4$ and so decreases with increasing photon wavelength. Rayleigh scattering can be neglected in most atmospheres, but it is important in the UV for the extended envelopes of supergiant stars, and in cool main-sequence stars. The scattering of photons from small particles is also responsible for the reddening of starlight as it passes through interstellar dust.

Example 2.4:
The energy of an electron in the n = 2 orbit of a hydrogen atom is given by:

$$E_2 = -\frac{13.6}{2^2}eV = -3.4eV$$

A photon must have an energy of at least $\chi_2 = 3.40$ eV to eject this electron from the atom. Thus any photon with a wavelength smaller than 364.7 nm is capable of ionizing a hydrogen atom in the first excited state (n = 2). The opacity of the stellar material suddenly increases at wavelengths $\lambda \leq 364.7nm$, and the radiative flux measured for the star accordingly decreases. The abrupt drop in the continuous spectrum of a star at this wavelength, called the **Balmer jump**, is evident in the Sun's spectrum (Fig. 5). **The size of the Balmer jump in hot stars depends on the fraction of hydrogen atoms that are in the first excited state. This fraction is determined by the temperature via the Boltzmann equation, so a measurement of the size of the Balmer jump can be used to determine the temperature of the atmosphere.** For cooler or very hot stars with other significant sources of opacity, the analysis is more complicated, but the size of the Balmer jump can still be used as a probe of atmospheric temperatures.

The effect of line blanketing affects the measured color indices, making the star appear more red than a model blackbody star of the same effective temperature, and thus increasing the values of both $U - B$ and $B - V$.

9.2.5 Continuum Opacity and the H^- Ion

The primary source of the continuum opacity in the atmospheres of stars later than F0 is the photoionization of H^- ions. An H^- ion is a hydrogen atom that possesses an extra electron. Because of the partial shielding that the nucleus provides, a second electron can be loosely bound to the atom on the side of the ion opposite that of the first electron. In this position the second electron is closer to the positively charged nucleus than it is to the negatively charged electron. Therefore, according to Coulomb's law, the net force on the extra electron is attractive.(真绕)

The binding energy of the H^- ion is only 0.754 eV, compared with the 13.6 eV required to ionize the ground state hydrogen atom. As a result, any photon with energy in excess of the ionization energy can be absorbed by an H^- ion, liberating the extra electron; the remaining energy becomes kinetic energy. Conversely, an electron captured by a hydrogen atom to form H^- will release a photon corresponding to the kinetic energy lost by the electron together with the ion's binding energy

$$H + e^- \rightleftharpoons H^- + \gamma$$

Since 0.754 eV corresponds to a photon with a wavelength of 1640 nm, any photon with a wavelength less than that value can remove an electron from the ion (bound–free opacity). At longer wavelengths, H^- can also contribute to the opacity through free–free absorption. Consequently, H^- ions are an important source of continuum opacity for stars cooler than F0. However, the H^- ions become increasingly ionized at higher temperatures and therefore make less of a contribution to the continuum opacity. For stars of spectral types B and A, the photoionization of hydrogen atoms and free–free absorption are the main sources of the continuum opacity. At the even higher temperatures encountered for O stars, the ionization of atomic hydrogen means that electron scattering becomes more and more important, with the photoionization of helium also contributing to the opacity.

Molecules can survive in cooler stellar atmospheres and contribute to the bound–bound and bound–free opacities; the large number of discrete molecular absorption lines is an efficient impediment to the flow of photons. Molecules can also be broken apart into their constituent atoms by the absorption of photons in the process of photodissociation(光致分解), which plays an important role in planetary atmospheres.

The total opacity is the sum of the opacities due to all of the preceding sources:

$$\kappa_\lambda = \kappa_{\lambda,bb} + \kappa_{\lambda,bf} + \kappa_{\lambda,ff} + \kappa_{es} + \kappa_{H^-}$$

(the H^- opacity is explicitly included because of its unique and critical contribution to the opacity in many stellar atmospheres, including our Sun).
The total opacity depends not only on the wavelength of the light being absorbed but also on the composition, density, and temperature of the stellar material.

The additional dependencies of the opacity on the electron number density, states of excitation and ionization of the atoms and ions, and other factors can all be calculated from the composition, density, and temperature.

9.2.6 The Rosseland Mean Opacity

It is often useful to employ an opacity that has been averaged over all wavelengths (or frequencies) to produce a function that depends only on the composition, density, and temperature. Although a variety of different schemes have been developed to compute a wavelength-independent opacity, by far the most commonly used is the **Rosseland mean opacity**, often simply referred to as the **Rosseland mean**.

9.3 RADIATIVE TRANSFER

In an equilibrium, steady-state star, there can be no change in the total energy contained within any layer of the stellar atmosphere or interior

9.3.1 Photon Emission Processes

9.3.2 The Random Walk

A more careful analysis (performed in Section 4) shows that the average level in the atmosphere from which photons of wavelength A escape is at a characteristic optical depth of about $\tau_\lambda = 2/3$. Looking into a star at any angle, we always look back to an optical depth of about $\tau_\lambda = 2/3$, as measured straight back along the line of sight. In fact, a star's photosphere is defined as the layer from which its visible light originates—that is, where $\tau_\lambda \approx 2/3$ for wavelengths in the star's continuum.

The realization that an observer looking vertically down on the surface of a star sees photons from $\tau_\lambda \approx 2/3$ offers an important insight into the formation of spectral lines.

9.3.3 Limb Darkening

Another implication of receiving radiation from an optical depth of about two-thirds is shown in Fig. 9.2. The line of sight of an observer on Earth viewing the Sun is vertically downward at the center of the Sun's disk but makes an increasingly larger angle θ with the vertical near the edge, or limb, of the Sun. Looking near the limb, the observer will not see as deeply into the solar atmosphere and will therefore see a lower temperature at an optical depth of two-thirds (compared to looking at the center of the disk)(越靠近中心, 温度越高, 因此同样是 2/3, 靠近边缘的看的更浅, 所以也更暗). As a result, the limb of the Sun appears darker than its center. This **limb darkening** has been observed in the light curves of some eclipsing binaries. More detailed information on limb darkening may be found later in this section.

9.3.4 The Radiation Pressure Gradient

Because the temperature in a star decreases outward, the radiation pressure is smaller at greater distances from the center

9.4 THE TRANSFER EQUATION

In this section, we will focus on a more thorough examination of the flow of radiation through a stellar atmosphere. (Although the focus of this discussion is on stellar atmospheres, much of the discussion is applicable to other environments as well, such as light traversing an interstellar gas cloud.) We will develop and solve the basic equation of radiative transfer using several standard assumptions. In addition, we will derive the variation of temperature with optical depth in a simple model atmosphere before applying it to obtain a quantitative description of limb darkening

9.4.1 The Emission Coefficient

9.4.2 The Source Function and the Transfer Equation

The ratio of the emission coefficient to the absorption coefficient is called the **source function** $S_\lambda \equiv j_\lambda/\kappa_\lambda$. **It describes how photons originally traveling with the beam are removed and replaced by photons from the surrounding gas.**
Therefore, in terms of the source function,

$$-\frac{1}{\kappa_\lambda \rho}\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$

This is one form of the **equation of radiative transfer** (usually referred to as the **transfer equation**)

On the other hand, if the intensity is less than the source function, the intensity increases with distance. This is merely a mathematical restatement of the tendency of the photons found in the beam to resemble the local source of photons in the surrounding gas. Thus the intensity of the light tends to become equal to the local value of the source function, although the source function itself may vary too rapidly with distance for an equality to be attained.

9.4.3 The Special Case of Blackbody Radiation

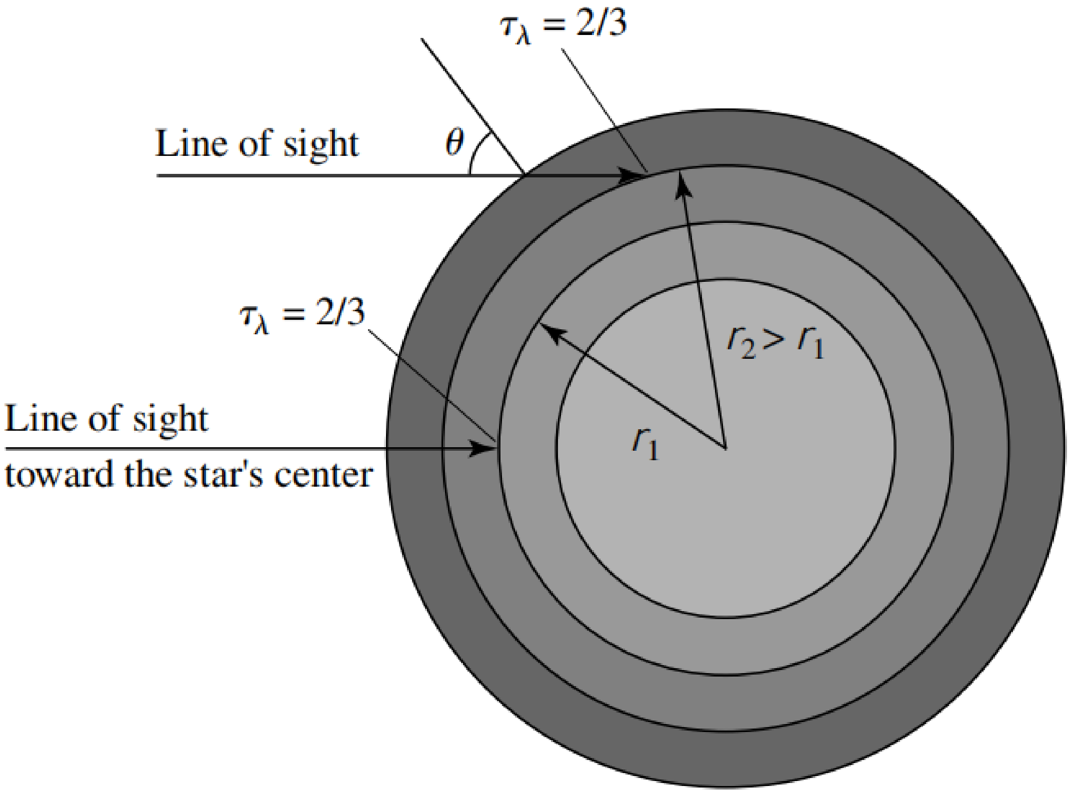

Figure 9.2: Limb darkening. The distance traversed within the atmosphere of the star to reach a specified radial distance r from the star's center increases along the line of sight of the observer as θ increases. This implies that to reach a specified optical depth (e.g., $\tau_\lambda = 2/3$), the line of sight terminates at greater distances (and cooler temperatures) from the star's center as θ increases. Note that the physical scale of the photosphere has been greatly exaggerated for illustration purposes. The thickness of a typical photosphere is on the order of 0.1% of the stellar radius